Hochschild cohomology and Gerstenhaber bracket of twisted tensor product algebras

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Setup

Definition

The *Hochschild cohomology* of a k algebra A is $HH^*(A) = \mathrm{Ext}^*_{A^e}(A, A)$.

Definition (Čap, Schichl, Vanžura)

The *twisted tensor product* $A \otimes_\tau B$ of A and B via $\tau : B \otimes A \longrightarrow A \otimes B$ is $A \otimes B$ with multiplication $m_\tau = (m_A \otimes m_B) \circ (1 \otimes \tau \otimes 1)$.

Goal

Understand $HH^*(A \otimes_\tau B)$ in terms of $HH^*(A)$ and $HH^*(B)$.

Given a resolution of A as A^e module, and a resolution of B as B^e module, we compute a resolution of $A \otimes_\tau B$ as $(A \otimes_\tau B)^e$ module.

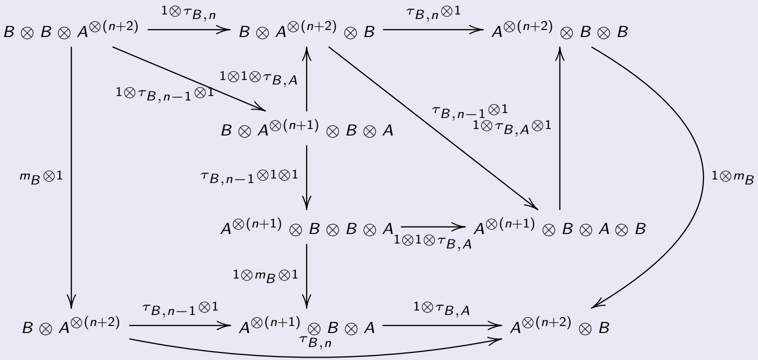
Outline

Extension of ideas by Grimley, Negron, Nguyen, Shepler, and Witherspoon:

i. The bar resolutions of A and B are *compatible* with τ.
ii. There is a chain map isomorphism lifting the *twisted module structure* on these resolutions.
iii. Construct the Gerstenhaber bracket from *contracting homotopies*.
iv. These results *descend* to the Koszul resolution.

Techniques and Results

Technique(s): fancy diagram chasing



Examples (Grimley, Lopes, Nguyen, Shirikov, Solotar, Witherspoon)

For some $q \in k^*$, $k\langle x, y\rangle/(x^2, y^2, xy + qyx)$, and $k\langle x, y\rangle/(xy - yx - y^2)$.

Thank you!