Hochschild Cohomology of Twisted Tensor Product Algebras

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Setup

Definition

The *Hochschild cohomology* of a k algebra A is $HH^*(A) = \mathrm{Ext}^*_{A^e}(A, A)$.

Definition (Čap, Schichl, Vanžura)

The *twisted tensor product* $A \otimes_\tau B$ of A and B via $\tau : B \otimes A \longrightarrow A \otimes B$ is $A \otimes B$ with multiplication $m_\tau = (m_A \otimes m_B) \circ (1 \otimes \tau \otimes 1)$.

Goal

Understand $HH^*(A \otimes_\tau B)$ in terms of $HH^*(A)$ and $HH^*(B)$.

Ideas

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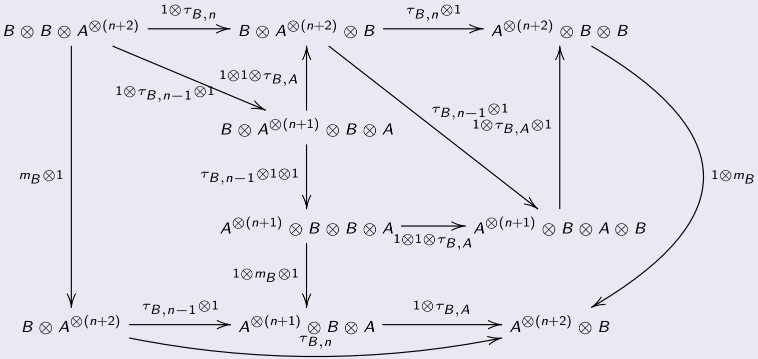
1. Given a resolution of A as A^e module, and a resolution of B as B^e module, we compute a resolution of $A \otimes_\tau B$ as $(A \otimes_\tau B)^e$ module.
2. We will need these resolutions to be *compatible* with τ. (Shepler, Witherspoon)

Techniques

Fancy diagram chasing



Homotopy lifting

Requires a resolution, a diagonal map, and a cocycle.

Applications (Grimley, Negron, Nguyen, Shirikov, Witherspoon)

For some $q \in k^*$, $k\langle x, y\rangle/(x^2, y^2, xy + qyx)$, and $k\langle x, y\rangle/(xy - yx - y^2)$.

Twisting by a bicharacter: $A \otimes_\tau B = A \otimes^t B$

Let A, B be k-algebras graded by the commutative groups F, G respectively, let $t : F \otimes_{\mathbb{Z}} G \to k^\times$ be a bicharacter. Then $\tau(b \otimes a) = t(|a|, |b|)a \otimes b$ is a twisting map. Then (GNW, OOW):

$$HH^{*,F'\oplus G'}(A \otimes^t B) \cong HH^{*,F'}(A) \otimes HH^{*,G'}(B).$$