NAME AND UCLA ID:

Task 1: Read Sections 19, 20, and 24.

Exercise 1: Let S be a G-set, let $s_1, s_2 \in S$ and $x \in G$ satisfy $s_1 = xs_2$, prove that $G_{s_1} = xG_{s_2}x^{-1}$.

Exercise 2: Let G be a group and S a non-empty set. Show that:

1. If $\star : G \times S \to S$ is a G-action then $\varphi : G \to \Sigma(S)$ defined by $\varphi(x)(s) = x \star s$ (namely $\varphi(x) = \varphi_x : S \to S$ with $\varphi_x(s) = x \star s$) is a group homomorphism. It is called the permutation representation of G on S.
2. If $\varphi : G \to \Sigma(S)$ is a group homomorphism, then $\star : G \times S \to S$ defined by $x \star s = \varphi_x(s)$ where $\varphi_x = \varphi(x) : S \to S$ is a G-action.

Exercise 3: Let $H \subseteq G$ be a subgroup, consider the equivalence relation on G given by $a \equiv b \mod H$ whenever $ab^{-1} \in H$ (this gives right cosets). Find a group A, a set S, and a left A-action on S such that the equivalence classes of the A-action are the right cosets of H in G.

Exercise 4: Compute all the conjugacy classes and isotropy subgroups of A_4 (Section 24 may be helpful).

Exercise 5: Let G be a group of order p^n for $p \in \mathbb{Z}^+$ a prime. Prove that if the center of G has order at least p^{n-1}, then G is abelian.

Exercise 6: Let G be a finite group and $p \in \mathbb{Z}^+$ be the smallest prime dividing the order of G. If H is a subgroup of G of index p, show that $H \trianglelefteq G$.

Exercise 7: Let G be a group and H a subgroup of finite index. Show that H contains a subgroup N that is normal and of finite index in G.