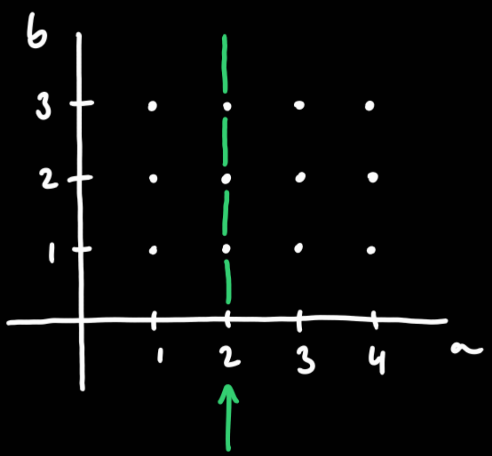
Remark 2.9.: Let $\{m<1 \mid m\in\mathbb{Z}^+\}$. This is empty.

This tries to convey that we still need to prove that $P(1)$ is true when using the "Second Principle of Finite Induction".

Assume $P(m)$ is true for $m<n$, then $P(n)$ is true.

Doing this with $n=1$ gives $\{m<1 \mid m\in\mathbb{Z}^+\}=\emptyset$.

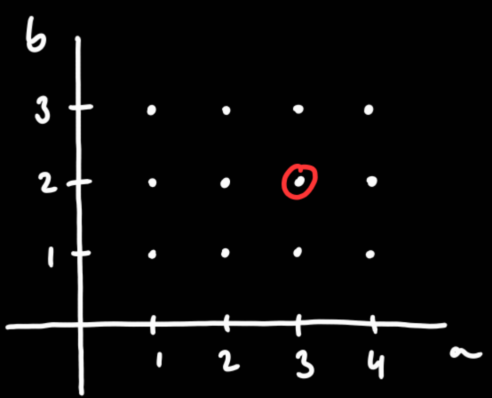
Claim 2.12.: $a=n,\ b=m$



$a=2=3-1,\ N=3.$

Step 1: $n=N-1$ $\quad a=N-1$

The claim holds for fixed $n=N-1$ and all m.



Step 2: $n=N$ $\quad m=M$ $\quad a=N$ $\quad b=M$

The claim holds for fixed $n=N$ and $m=M$.

$a=3=N,\ b=2=M$

Corollary 2.13.: There exist infinitely many n consecutive composite positive integers

integers.

Proof: Let $m \in \mathbb{Z}^+$ and $N = m(m+1)\cdots(m+n)$. Then $(n+1)! \mid N$. This means that $N = c\cdot(n+1)!$ for some $c \in \mathbb{Z}^+$. Pick any $2 \leq s \leq n+1$, then:

$$N+s = c\cdot(n+1)! + s = \left(c\cdot(n+1)\cdots(s+1)(s-1)\cdots 2\cdot 1 + 1\right)\cdot s.$$

$$(n+1)! = (n+1)\cdots s \cdots 2\cdot 1$$

Hence $s \mid N+s$. This is true for all $2 \leq s \leq n+1$, so:

$$\underbrace{N+2}_{\text{divisible by } 2}, N+3, \ldots, \underbrace{N+n+1}_{\text{divisible by } n+1} \quad \text{are all composite.}$$

$\square$.

Corollary 2.14.:

$$\binom{m}{n} := \frac{m\cdot(m-1)\cdots(m-n+1)}{n!}$$

multiplication of n consecutive positive integers, since $n \leq m$.

$$= \frac{(m-n+1)\cdot(m-n+2)\cdots(m-n+(n-1))\cdot(m-n+n)}{n!} = \frac{(m-n+1)_n}{n!}$$

Definition: $(m)_n := m\cdot(m+1)\cdots(m+n-1)$

Trick on 4.14.:

$$c = cax + cby \underset{\uparrow}{=} cax + ady = a\cdot(c\cdot x + dy) \quad \text{so } a \mid c.$$

$a \mid cb$ so $cb = a\cdot d$ for some $d \in \mathbb{Z}^+$

Definition: $p^e \| n$, then $p^e \mid n$ but $p^{e+1} \nmid n$.

$n = p_1^{e_1} \cdots p_r^{e_r}$, $1 < p_1 < \cdots < p_r$ prime , $e_1, \ldots, e_r \in \mathbb{Z}^+$.

$p^e \| n$ if and only if $p = p_i$ and $e = e_i$.