Example 2.20:

To solve:

$$\begin{aligned} 2x - 2y + 8z &= 9 \\ -2x + 2y + z &= 3 \\ x + 2y - 3z &= 8 \end{aligned}$$

we put this in the augmented matrix:

$$\left[\begin{array}{ccc|c} 2 & -2 & 8 & 9 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array}\right]$$

and we then pivot about a_{11}:

$$\left[\begin{array}{ccc|c} 2 & -2 & 8 & 9 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & -1 & 4 & \frac{9}{2} \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array}\right] \xrightarrow{R_2 + 2\cdot R_1} \left[\begin{array}{ccc|c} 1 & -1 & 4 & \frac{9}{2} \\ 0 & 0 & 9 & 12 \\ 1 & 2 & -3 & 8 \end{array}\right] \cdots$$

$$\cdots \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|c} 1 & -1 & 4 & \frac{9}{2} \\ 0 & 0 & 9 & 12 \\ 0 & 3 & -7 & \frac{7}{2} \end{array}\right]$$

. We now pivot about a_{32}:

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & \frac{9}{2} \\ 0 & 0 & 9 & 12 \\ 0 & 3 & -7 & \frac{7}{2} \end{array}\right] \xrightarrow{\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & -1 & 4 & \frac{9}{2} \\ 0 & 0 & 9 & 12 \\ 0 & 1 & -\frac{7}{3} & \frac{7}{6} \end{array}\right] \xrightarrow{R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{5}{3} & \frac{17}{3} \\ 0 & 0 & 9 & 12 \\ 0 & 1 & -\frac{7}{3} & \frac{7}{6} \end{array}\right]$$

we now pivot about a_{23}:

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{5}{3} & \frac{17}{3} \\ 0 & 0 & 9 & 12 \\ 0 & 1 & -\frac{7}{3} & \frac{7}{6} \end{array}\right] \xrightarrow{\frac{1}{9}R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{5}{3} & \frac{17}{3} \\ 0 & 0 & 1 & \frac{4}{3} \\ 0 & 1 & -\frac{7}{3} & \frac{7}{6} \end{array}\right] \xrightarrow{R_1 - \frac{5}{3}R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{31}{9} \\ 0 & 0 & 1 & \frac{4}{3} \\ 0 & 1 & -\frac{7}{3} & \frac{7}{6} \end{array}\right] \cdots$$

$$\cdots \xrightarrow{R_3 + \frac{7}{3}R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{31}{9} \\ 0 & 0 & 1 & \frac{4}{3} \\ 0 & 1 & 0 & \frac{77}{18} \end{array}\right]$$

and we finally exchange row 2 and row 3 to put this in row-reduced echelon form:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{31}{9} \\ 0 & 0 & 1 & \frac{4}{3} \\ 0 & 1 & 0 & \frac{77}{18} \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{31}{9} \\ 0 & 1 & 0 & \frac{77}{18} \\ 0 & 0 & 1 & \frac{4}{3} \end{array}\right]$$

where we can read:

$$\boxed{\begin{aligned} x &= \frac{31}{9} \\ y &= \frac{77}{18} \\ z &= \frac{4}{3} \end{aligned}}$$

, the solutions to the system.

Example 2.23:

Set x the number of dozens of A3 notebooks produced per day.
y A4
z A5

The table states that College station works $9x+12y+15z$ minutes per day.
Galveston $22x+24y+28z$
Qatar $6x+8y+8z$

Since we are told the maximum amount of minutes that each campus can work, we have:

$$\begin{aligned} 9x+12y+15z&=80\\ 22x+24y+28z&=160\\ 6x+8y+8z&=48 \end{aligned}$$ with augmented matrix: $\left[\begin{array}{ccc|c} 9 & 12 & 15 & 80\\ 22 & 24 & 28 & 160\\ 6 & 8 & 8 & 48 \end{array}\right]$.

Pivoting about a_{11} we obtain:

$$\left[\begin{array}{ccc|c} 9 & 12 & 15 & 80\\ 22 & 24 & 28 & 160\\ 6 & 8 & 8 & 48 \end{array}\right] \xrightarrow{\frac{1}{9}R_1} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{80}{9}\\ 22 & 24 & 28 & 160\\ 6 & 8 & 8 & 48 \end{array}\right] \xrightarrow{R_2-22\cdot R_1} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{80}{9}\\ 0 & \frac{-16}{3} & \frac{-26}{3} & -\frac{320}{9}\\ 6 & 8 & 8 & 48 \end{array}\right] \cdots$$

$$\cdots \xrightarrow{R_3-6\cdot R_1} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{80}{9}\\ 0 & \frac{-16}{3} & \frac{-26}{3} & -\frac{320}{9}\\ 0 & 0 & -2 & \frac{-16}{3} \end{array}\right]$$, then pivoting about a_{22} we obtain:

$$\left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{80}{9}\\ 0 & -\frac{16}{3} & \frac{-26}{3} & -\frac{320}{9}\\ 0 & 0 & -2 & \frac{-16}{3} \end{array}\right] \xrightarrow{\frac{-3}{16}\cdot R_2} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{80}{9}\\ 0 & 1 & \frac{13}{8} & \frac{20}{3}\\ 0 & 0 & -2 & -\frac{16}{3} \end{array}\right] \xrightarrow{R_1-\frac{4}{3}R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{-1}{2} & 0\\ 0 & 1 & \frac{13}{8} & \frac{20}{3}\\ 0 & 0 & -2 & \frac{-16}{3} \end{array}\right].$$

Then pivoting about a_{33} we obtain:

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{-1}{2} & 0\\ 0 & 1 & \frac{13}{8} & \frac{20}{3}\\ 0 & 0 & -2 & -\frac{16}{3} \end{array}\right] \xrightarrow{\frac{-1}{2}\cdot R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{-1}{2} & 0\\ 0 & 1 & \frac{13}{8} & \frac{20}{3}\\ 0 & 0 & 1 & \frac{8}{3} \end{array}\right] \xrightarrow{R_2-\frac{13}{8}R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{-1}{2} & 0\\ 0 & 1 & 0 & \frac{7}{3}\\ 0 & 0 & 1 & \frac{8}{3} \end{array}\right]$$

$$\xrightarrow{R_1+\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{4}{3}\\ 0 & 1 & 0 & \frac{7}{3}\\ 0 & 0 & 1 & \frac{8}{3} \end{array}\right]$$, where we can read: $\boxed{x=\frac{4}{3},\ y=\frac{7}{3},\ z=\frac{8}{3}}$ the solution to the system.

Example 2.28.: PLEASE TRY THIS EXAMPLE ON YOUR OWN.

Example 2.29.: a) P has size $\boxed{3\times 4}$.

b) a_{24} has value $\boxed{0}$. It means that Edinburgh does not produce Stanmore 2 last May. (did)

c) $320+280+460+280=\boxed{1340}$ It means that Dundee produced a total of 1340 loudspeakers last May.

d) $280+0+880=\boxed{1160}$. It means that the company Marshall produced a total of 1160 Stanmore 2 loudspeakers last May.

Example 2.33.: I will only do a few, the rest are done similarly.

1. $$A+B=\begin{bmatrix}4&-5&6\\-11&2&4\\3&6&8\end{bmatrix}+\begin{bmatrix}-3&5&1\\4&15&8\\2&9&-1\end{bmatrix}=\begin{bmatrix}4-3&-5+5&6+1\\-11+4&2+15&4+8\\3+2&6+9&8-1\end{bmatrix}=\begin{bmatrix}1&0&7\\-7&17&12\\5&15&7\end{bmatrix}.$$

2. $$B+A=\begin{bmatrix}-3&5&1\\4&15&8\\2&9&-1\end{bmatrix}+\begin{bmatrix}4&-5&6\\-11&2&4\\3&6&8\end{bmatrix}=\begin{bmatrix}-3+4&5-5&1+6\\4-11&15+2&8+4\\2+3&9+6&-1+8\end{bmatrix}=\begin{bmatrix}1&0&7\\-7&17&12\\5&15&7\end{bmatrix}.$$

3. $$C+D=\begin{bmatrix}-8&7&12\\5&-21&3\\-9&18&-6\end{bmatrix}+\begin{bmatrix}9&-7&-5\\-12&38&9\\14&-3&13\end{bmatrix}=\begin{bmatrix}-8+9&7-7&12-5\\5-12&-21+38&3+9\\-9+14&18-3&-6+13\end{bmatrix}=\begin{bmatrix}1&0&7\\-7&17&12\\5&15&7\end{bmatrix}.$$

4. $$C+D=\begin{bmatrix}9&-7&-5\\-12&38&9\\14&-3&13\end{bmatrix}+\begin{bmatrix}-8&7&12\\5&-21&3\\-9&18&-6\end{bmatrix}=\begin{bmatrix}9-8&-7+7&-5+12\\-12+5&38-21&9+3\\14-9&-3+18&13-6\end{bmatrix}=\begin{bmatrix}1&0&7\\-7&17&12\\5&15&7\end{bmatrix}.$$

5. We have $A+B=C+D$.

Example 2.37.: We do a variation, finding X satisfying $2X+B^T=3A$. We have:

$2X+B^T=3A$ if and only if $2X=3A-B^T$ if and only if $X=\frac{3}{2}A-\frac{1}{2}\cdot B^T$.

We then compute:
$$\boxed{X}=\frac{3}{2}\cdot\begin{bmatrix}3&4\\-1&2\end{bmatrix}-\frac{1}{2}\cdot\begin{bmatrix}3&2\\-1&2\end{bmatrix}^T=\frac{3}{2}\cdot\begin{bmatrix}3&4\\-1&2\end{bmatrix}-\frac{1}{2}\cdot\begin{bmatrix}3&-1\\2&2\end{bmatrix}=$$

PLEASE TRY THIS ON YOUR OWN. ($2X+B=3A$).

$$=\begin{bmatrix}\frac{9}{2}&6\\-\frac{3}{2}&3\end{bmatrix}-\begin{bmatrix}\frac{3}{2}&\frac{-1}{2}\\1&1\end{bmatrix}=\begin{bmatrix}\frac{9}{2}-\frac{3}{2}&6+\frac{1}{2}\\-\frac{3}{2}-1&3-1\end{bmatrix}=\boxed{\begin{bmatrix}3&\frac{13}{2}\\-\frac{5}{2}&2\end{bmatrix}}$$

Example 2.40.:

$$AB = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 4 & -1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 3\cdot 1 + 1\cdot 4 + 4\cdot 2 & 3\cdot 3 + 1\cdot(-1) + 4\cdot 4 \\ 1\cdot 1 + 2\cdot 4 + 3\cdot 2 & 1\cdot 3 + 2\cdot(-1) + 3\cdot 4 \end{bmatrix} = \begin{bmatrix} 15 & 24 \\ 15 & 13 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 3 \\ 4 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 & 4 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1\cdot 3 + 3\cdot 1 & 1\cdot 1 + 3\cdot 2 & 1\cdot 4 + 3\cdot 3 \\ 4\cdot 3 + (-1)\cdot 1 & 4\cdot 1 + (-1)\cdot 2 & 4\cdot 4 + (-1)\cdot 3 \\ 2\cdot 3 + 4\cdot 1 & 2\cdot 1 + 4\cdot 2 & 2\cdot 4 + 4\cdot 3 \end{bmatrix} = \begin{bmatrix} 6 & 7 & 13 \\ 11 & 2 & 13 \\ 10 & 10 & 20 \end{bmatrix}.$$

Notice that AB, of size 2×2, and BA, of size 3×3, do <u>not</u> have the same size. In particular, even when multiplications are defined, they are <u>not</u> commutative:

$AB \neq BA$.

Example 2.41.: PLEASE TRY THIS EXAMPLE ON YOUR OWN.