Math 141 Exam 2 – Fall 2019 – Version A

Instructor: P. Ocal

DIRECTIONS:

1. TURN OFF cell phones and smart watches and put them away.
2. There are two sections to this exam, a multiple-choice section and a workout section. Problems #1-10 are multiple choice worth 4 points each and #11-16 are workout problems with the point value given next to each problem.
   - For the multiple choice, record your answer on the exam. No partial credit will be given for this section of the exam.
   - For the workout portion you must show appropriate, legible, and meaningful work to receive full credit. If you do a majority of the work on your calculator, then you must write down the features or functions you used on the calculator to solve the problem. If you choose to just write down an answer without any supported work, you will NOT receive full credit. Please CLEARLY indicate your answers for all problems in the workout section.
3. You may use a TI-83, TI-84 or TI-Nspire (non-CAS) graphing calculator on this exam, but the memory must be cleared before starting this exam.
4. You have 75 minutes to take this exam.

Helpful Information:
(a) The alphabet has 26 letters; a through z.
(b) Standard Deck of Cards: 13 ranks (ace, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K) and 4 suits (diamonds, hearts, clubs, spades)

$A\diamondsuit$	$2\diamondsuit$	$3\diamondsuit$	$4\diamondsuit$	$5\diamondsuit$	$6\diamondsuit$	$7\diamondsuit$	$8\diamondsuit$	$9\diamondsuit$	$10\diamondsuit$	$J\diamondsuit$	$Q\diamondsuit$	$K\diamondsuit$
$A\heartsuit$	$2\heartsuit$	$3\heartsuit$	$4\heartsuit$	$5\heartsuit$	$6\heartsuit$	$7\heartsuit$	$8\heartsuit$	$9\heartsuit$	$10\heartsuit$	$J\heartsuit$	$Q\heartsuit$	$K\heartsuit$
$A\clubsuit$	$2\clubsuit$	$3\clubsuit$	$4\clubsuit$	$5\clubsuit$	$6\clubsuit$	$7\clubsuit$	$8\clubsuit$	$9\clubsuit$	$10\clubsuit$	$J\clubsuit$	$Q\clubsuit$	$K\clubsuit$
$A\spadesuit$	$2\spadesuit$	$3\spadesuit$	$4\spadesuit$	$5\spadesuit$	$6\spadesuit$	$7\spadesuit$	$8\spadesuit$	$9\spadesuit$	$10\spadesuit$	$J\spadesuit$	$Q\spadesuit$	$K\spadesuit$

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

Student's Signature: Answer Key

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Multiple Choice (40 pts)	#11 (12 pts)	#12 (6 pts)	#13 (8 pts)	#14 (12 pts)	#15 (10 pts)	16 (12 pts)	Total (100 pts)

Circle the corresponding answer on your exam. No partial credit will be given. Each multiple choice problem is worth 4 points.

Question 1. Consider a universal set U consisting of 17 elements. Given that $n(A) = 12$, $n(B) = 8$, and $n(A^c \cup B) = 10$, determine the number of elements in $A^c \cap B$.

(a) $n(A^c \cap B) = 3$

(b) $n(A^c \cap B) = 7$

(c) $n(A^c \cap B) = 12$

(d) $n(A^c \cap B) = 8$

(e) $n(A^c \cap B) = 10$

$n(U) = 17 \qquad n(A^c) = n(U) - n(A) = 5$

$n(A^c \cup B) = n(A^c) + n(B) - n(A^c \cap B)$

$n(A^c \cap B) = n(A^c) + n(B) - n(A^c \cup B) =$

$= 5 + 8 - 10 = 3$

Question 2. A shelf in a library contains three identical mathematics, three identical social science, and two identical biology books. How many distinguishable ways can these eight books be arranged on a shelf?

(a) 5040

(b) 560

(c) 6720

(d) 1120

(e) 20160

$$\frac{8!}{3!\,3!\,2!} = 560$$

Question 3. Let S be the sample space of possible rolls of two fair four-sided die. Let A be the event where the sum of the dice values is an odd number. Calculate $n(A)$.

(a) $n(A) = 8$

(b) $n(A) = 6$

(c) $n(A) = 15$

(d) $n(A) = 30$

(e) $n(A) = 12$

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Question 4. Suppose for some universal set U, we know that A is a subset of B and neither A nor B are empty, determine which of the following must be true.

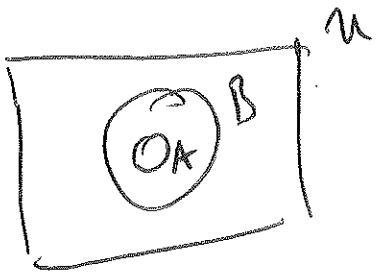
(a) $A \cup B = A$

(b) $A \cap B = \emptyset$

(c) $A \cap B = A$

(d) $A^c \subseteq B^c$

(e) $A^c \cap B^c = U$



Question 5. Suppose you have an unbounded feasible set in the first quadrant ($x \geq 0, y \geq 0$) with corner points $(10,0)$, $(2,4)$, $(6,2)$, and $(0,8)$. Find the minimum and maximum of the function $P = 3x + 4y$ on this feasible set.

(a) Maximum = 32 and minimum = 22.

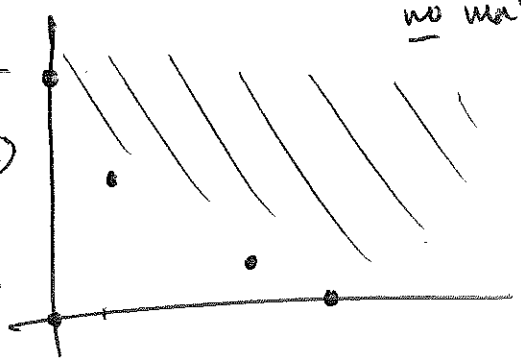
(b) Maximum = 32 and minimum does not exist.

(c) Maximum does not exist and minimum = 20.

(d) Maximum does not exist and minimum = 22.

(e) P does not have a maximum nor a minimum.

(x,y)	P
$(10,0)$	30
$(2,4)$	22
$(6,2)$	26
$(0,8)$	32



Question 6. A five-card sample is chosen from a standard deck of 52 cards. How many different five-card samples are possible if the sample must contain exactly two queens and exactly two kings?

(a) 6864

(b) 36

(c) 156

(d) 528

(e) 1584

$C(4,2) \cdot C(4,2) \cdot C(44,1)$

two queens · two kings · any other card (not queen nor king)

Question 7. Let $U = \{x | x \text{ is an even integer and } 0 \leq x \leq 15\}$, $A = \{x | x \in U, x \text{ is odd, and } x < 10\}$ and $B = \{2, 8, 14\}$. Find $A \cup B$.

(a) A

(b) B

(c) U

(d) $\emptyset$

(e) $\{0, 2, 4, 6, 8, 10\}$

$A = \emptyset$

$A \cup B = B$

Question 8. A Washington D.C. license plate consists of two capital letters followed by four digits. Determine the number of possible plates if all digits must be unique, but the letters may be repeated.

(a) $650 \cdot 10^4$

(b) $26^2 \cdot 10^4$

(c) $3,407,040$

(d) $3,276,000$

(e) $26^2 \cdot 10!$

26 26 — letters

10 9 8 7 — digits

Question 9. You have eight black socks, six red socks, and ten blue socks in your drawer. You are running late for a trip, so you pick out six socks at random without looking. How many combinations of six socks contain at least four red socks?

(a) 2295

(b) 2404

(c) 2348

(d) 1930

(e) 2403

4 red 5 red 6 red

$C(6,4)\cdot C(18,2) + C(6,5)\cdot C(18,1) + C(6,6) =$

Question 10. Let U be the set of all students enrolled at Texas AM University and let the sets A and B be defined as follows:

$$A = \{x \in U | x \text{ is taking an agriculture course}\} \text{ and } B = \{x \in U |\ x \text{ is taking a business course}\}$$

Which of the following gives the best description of the set $A^c \cup B$?

(a) It is the set of all students taking an agriculture and a business course.

(b) It is the set of all students not taking an agriculture course but who are taking a business course.

(c) It is the set of all students taking an agriculture or business course.

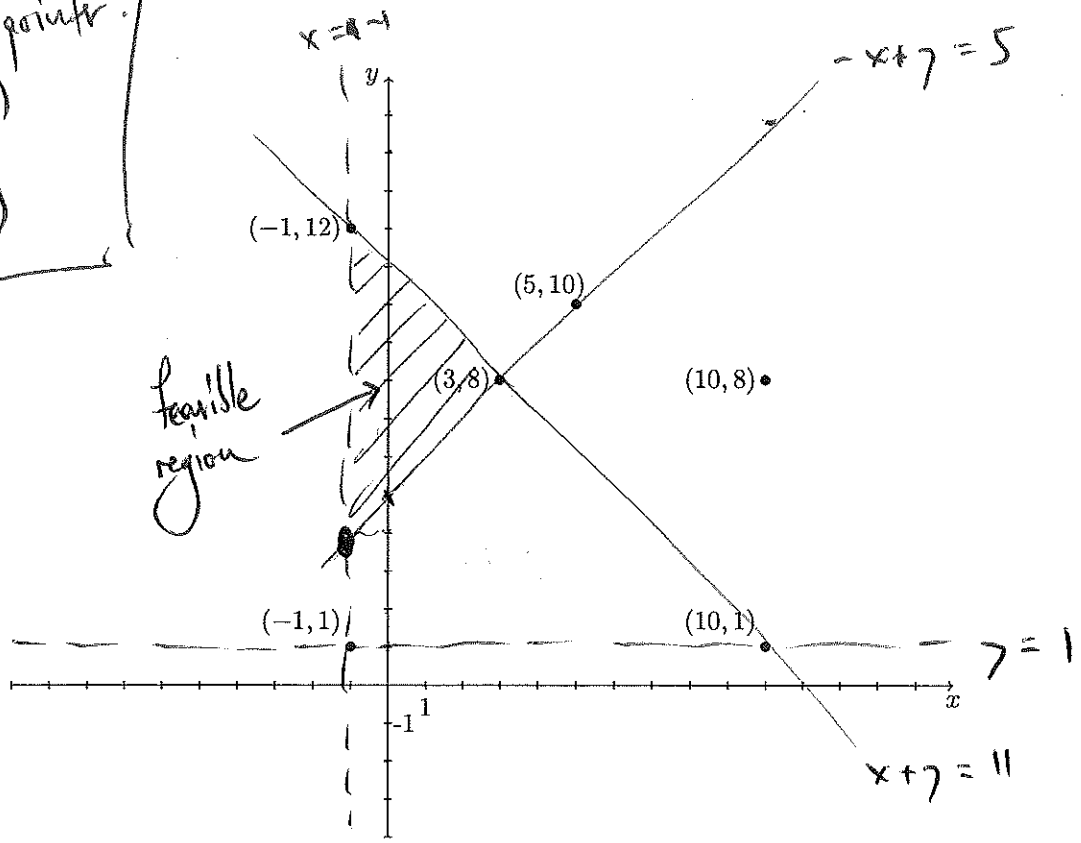
(d) It is the set of all students not taking an agriculture course or who are taking a business course.

(e) It is the set of all students taking neither an agriculture nor a business course.

Question 11. *(12 points)* Use the partially filled graph below to find the corner points of the feasible region subject to the constraints given below. Be sure to clearly label the feasible region and the corner points graph.

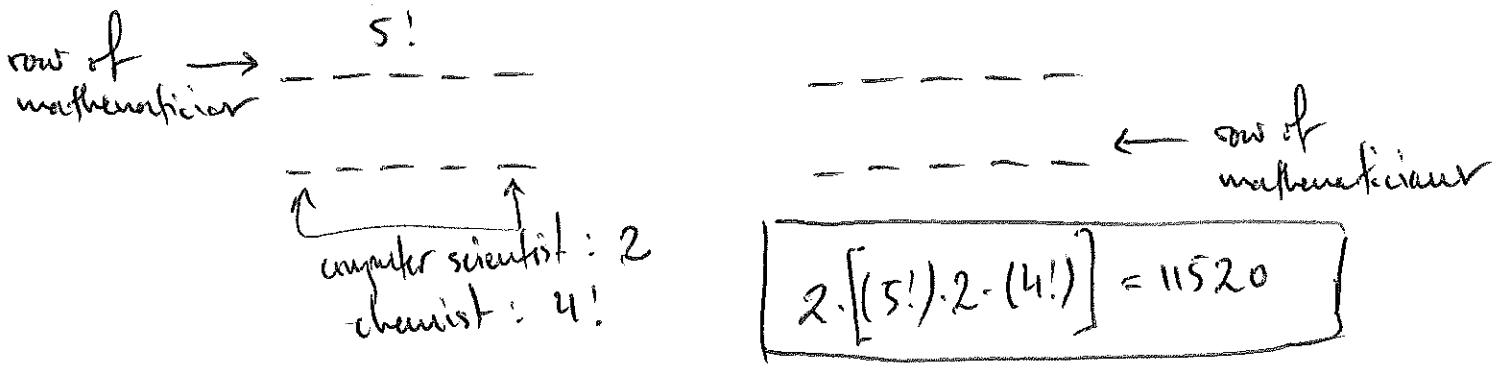
$$\begin{cases} x \geq -1 \\ y \geq 1 \\ x + y \leq 11 \\ -x + y \geq 5 \end{cases}$$

corner points:
(-1, 12)
(3, 8)
(-1, 4)

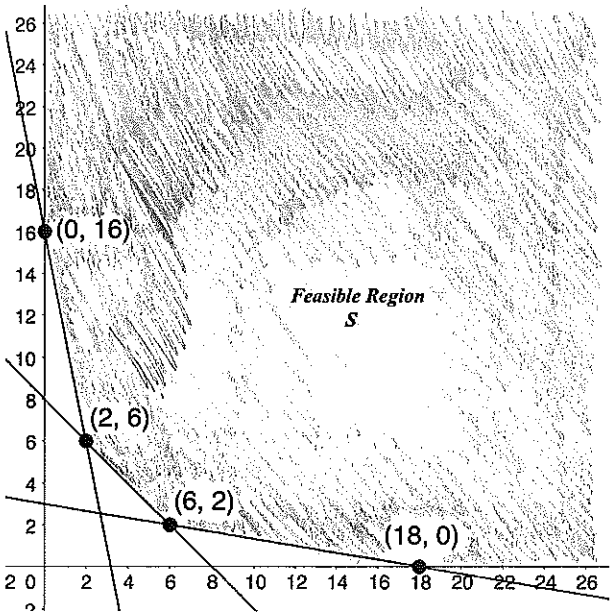


The corner points for the feasible region are: _____

Question 12. *(6 points)* An interdisciplinary congress has a an organizing committee comprised of ten experts; five mathematicians, one computer scientist, and four chemists. At the organizational meeting the committee is required to sit in two rows of five. If the five mathematicians must sit in the same row, and the single computer scientist must sit at the beginning or end of a row, determine how many different sitting arrangements are possible.



Question 13. Brazos Valley Trash Can Company manufacturers metal and plastic trash cans. It costs \$50 to produce each metal trash can (x) and \$10 to produce each plastic trash can (y). The graph below shows the unbounded feasible region, S, that results from graphing a system of five constraints. If the company is trying to minimize costs subject to these five constraints, find the objective function, the number of metal and plastic trash cans they should produce to obtain minimum production costs, and the corresponding minimum cost.



$C = 50 \cdot x + 10 \cdot y$

(x, y)	C
$(0, 16)$	160
$(2, 6)$	160
$(6, 2)$	320
$(18, 0)$	900

(a) *(2 points)* The objective function is $C = 50x + 10y$

(b) *(4 points)* How many of each trash can should the company produce to minimize costs?

(0 metal, 16 plastic) or (1 metal, 11 plastic) or (2 metal, 6 plastic)

(c) *(2 points)* What is the minimum cost?

\$160

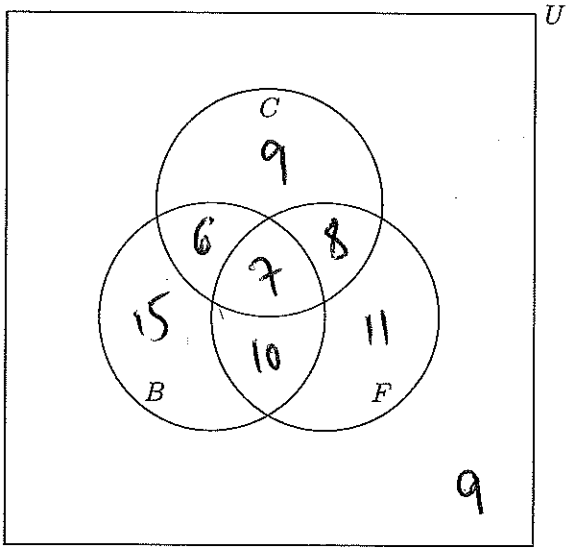
Question 14. Denote by U the set of the 75 students that form the Student Senate at Texas A&M University. Of those, denote:

- F as the set of the 36 freshmen students.
- B as the set of the 38 students who take the bus route *Revellie*.
- C as the set of the 30 students who are enrolled in chemistry courses.

We also know that

- 15 students take the bus but neither have chemistry courses nor are freshman.
- 10 freshman students take the bus, but do not have chemistry courses.
- 11 students are freshmen, but do not take the bus nor have chemistry classes.
- 8 freshman students have chemistry classes, but do not take the bus.

(a) *(8 points)* Fill in the following Venn diagram in such a way that it represents the above information. Do not label the individual regions.



(b) *(2 points)* Give the notation for the set of students that take the bus and have chemistry courses, but are not freshman.

$B \cap C \cap F^c$

(c) *(2 points)* Calculate the number of students belonging to $B^c \cap F^c \cap C^c$.

$n(B^c \cap F^c \cap C^c) = 9$

Question 15. A coin is flipped three times and it is noted whether it landed on heads or tails at each flip. Let A be the event that at least one flip resulted in a tail and B be the event the second flip resulted in a head.

(a) *(4 points)* Find the sample space for the experiment.

$\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

(b) *(4 points)* Describe the event A^c in words.

$A^c = \{HHH\}$, the event that no tails are recorded.

(c) *(2 points)* Are the events A^c and B mutually exclusive? Explain the reason for your answer.

$A^c \cap B = \{HHH\}$, they are not mutually exclusive

$B = \{HHH, HHT, THH, THT\}$

Question 16. At a local Subway sandwich shop, there are twelve different items from which you can choose to add to your sandwich. Five items are different varieties of meats, three are different types of cheese, and four are different vegetables.

(a) *(4 points)* How many different types of sandwiches are possible if you choose to put any six items on your sandwich?

$C(12,6) = 924$

(a) *(4 points)* How many different types of sandwiches are possible if you choose to put six items on your sandwich where the six items are two meats, two cheeses, and two vegetables?

$C(5,2) \cdot C(3,2) \cdot C(4,2) = 180$

(a) *(4 points)* How many different types of sandwiches are possible if you choose to put six items on your sandwich, but at least three of the items must be a meat?

3 meat: $C(5,3) \cdot C(7,3)$ + 4 meat: $C(5,4) \cdot C(7,2)$ + 5 meat: $C(5,5) \cdot C(7,1)$ $= 462$