MATH 141:502 – INSTRUCTOR: PABLO S. OCAL – EXAM 1 19 September 2019

NAME AND NETID:

Instructions:

1. You may use one of the approved calculators for the course. Besides this calculator, no other electronic device is allowed (unless explicit permission). These unauthorized electronic devices should be turned off and placed in your bag. Failure to do so will result in a loss of points and/or further penalties.

2. The first part of the exam consists of multiple choice questions. More than one answer may be correct. There is space provided for working out the problem, and partial credit will be given for reasonable working even if the question is not answered correctly.

3. The second part of the exam consists of long answer questions, where you need to present all your work neatly in the space provided, and box the final answer(s). You will be graded on both your final answer and the quality and detail of your work.

Aggie Code of Honor: An Aggie does not lie, cheat or steal or tolerate those who do.

SIGNED: Answer Key

The signature in the box above certifies that you have thoroughly read the instructions and agree to abide by them.

Question	Points
1	/3
2	/3
3	/3
4	/3
5	/3
6	/3
7	/3
8	/3
9	/5
10	/6
11	/6
12	/5
13	/4
Total	/50

Question 1. Calculate the values of x and y given that: *[3]*

$$5\begin{bmatrix}4 & -5\\ y & 2\end{bmatrix}^T + 3\begin{bmatrix}0 & 7x\\ 1 & y\end{bmatrix} = 2\begin{bmatrix}10 & y-1\\ -11 & 10\end{bmatrix}.$$

A. $x = 61/21$, $y = -27/5$.

B. $x = -2$, $y = 10$.

C. $x = 16/21$, $y = 10$.

D. $x = 41/21$, $y = -27$.

E. $x = -4/7$, $y = 10/3$.

$$5\begin{bmatrix}4 & 7\\ -5 & 2\end{bmatrix} + \begin{bmatrix}0 & 21x\\ 3 & 37\end{bmatrix} = \begin{bmatrix}20 & 27-2\\ -22 & 20\end{bmatrix}$$

$$57 + 21x = 27 - 2$$

$$10 + 37 = 20 \qquad 7 = \frac{10}{3}$$

Question 2. The cost in dollars for Firestone to manufacture x tires is given by $C(x) = mx + 1560$, and the selling price of each tire is \$180. If 520 tires must be produced and sold for Firestone to break-even, determine the value of m. *[3]*

A. $m = 3$.

B. $m = 177$.

C. $m = 15$.

D. $m = 142$.

E. $m = 180$.

$$R(x) = 180 \cdot x$$

$$180 \cdot 520 = m \cdot 520 + 1560$$

$$m = 177.$$

Question 3. Solve in x and y the following system of equations, and determine which of the following is true: *[3]*

$$3x + 2y = 4, \quad 2x + 7y = 8, \quad 9x + 6y = 12.$$

A. There are infinitely many solutions.

B. $x + y = 3/2$.

C. $x + y = 28/17$.

D. There are no solutions.

E. None of the above.

$3x+2y=4$ gives $9x+6y=12$ by multiplying by 3.

$$\left.\begin{aligned} 3x+2y &= 4 \\ 2x+7y &= 8 \end{aligned}\right\} \quad x = \frac{12}{17}, \quad y = \frac{16}{17}$$

Question 4. Let A, B, and C be matrices of sizes $a \times 3$, $b \times c$, and $5 \times d$ respectively. If the product ABC is of size 3×2, determine the values (a, b, c, d). *[3]*

A. $(3, 3, 2, 2)$.

B. $(4, 2, 3, 3)$.

C. $(3, 3, 5, 2)$.

D. $(3, 5, 4, 2)$.

E. $(2, 3, 3, 4)$.

$(a \times 3) \cdot (b \times c) \cdot (5 \times d) = 3 \times 2$ $\quad (a=3, b=3, c=5, d=2)$

Question 5. Texaco sells three grades of gas: regular, medium, and premium. The following table gives the price per liter that Alice and Bob paid:

Table 1	Regular	Medium	Premium
Alice	\$2.00	\$2.50	\$2.00
Bob	\$2.50	\$2.00	\$1.50

and the following table gives the amount (in liters) of each grade of gas they purchased:

Table 2	Regular	Medium	Premium
Alice	2	5	10
Bob	8	3	1

Seeing Table 1 as matrix A and Table 2 as a matrix B, determine which of the following is represented by entry $(1,1)$ of AB^T. *[3]*

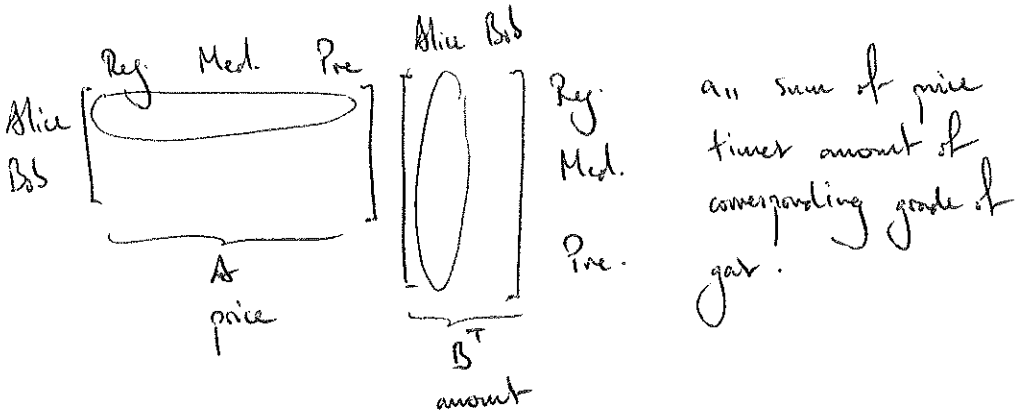
A. Nothing useful.

B. Total amount spent on gas grade regular.

(C.) Total amount Alice spent.

D. Total amount Bob spent.

E. Amount Bob spent on gas grade medium.



Question 6. The line L is associated with the linear function $f(x) = 5$. Determine which of the following is true. *[3]*

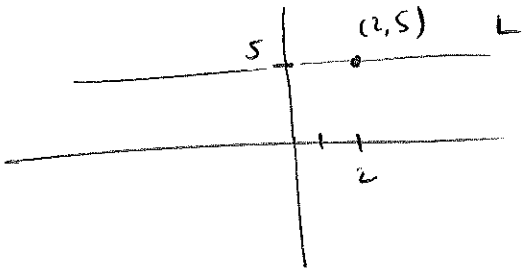
A. $(2,2)$ lies on L.

B. $(4,8)$ lies on L.

C. L is vertical.

(D.) L is horizontal.

(E.) $(2,5)$ lies on L.



Question 7. Determine which of the following provide a particular solution to the following reduced row echelon augmented matrix. *[3]*

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

A. $(-4, 2, 2)$.

B. $(-10, 0, 4)$.

C. $(-10, t, 1-t)$ for all $t \in \mathbb{R}$.

D. $(-10, 1-t, t)$ for all $t \in \mathbb{R}$.

E. $(-4, 4, 2)$.

$$\begin{cases} x + 3z = 2 \\ y + 2z = 8 \end{cases}$$

$$\begin{cases} -10 + 3 \cdot 4 = 2 \\ 0 + 2 \cdot 4 = 8 \end{cases}$$ So $(-10, 0, 4)$ is good.

$$\begin{cases} -4 + 3 \cdot 2 = 2 \\ 4 + 2 \cdot 2 = 8 \end{cases}$$ So $(-4, 4, 2)$ is good.

Question 8. Determine which of the following matrices are in reduced row echelon augmented matrix. *[3]*

A. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

B. $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -9 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -2 \end{array}\right]$.

C. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \end{array}\right]$. ← 2 needs to be 1.

D. $\left[\begin{array}{ccc|c} 1 & 5 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]$. ← second row needs to be last

E. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 \end{array}\right]$. ← second and third rows need to be exchanged.

Question 9. The weekly revenue in millions of dollars of HEB (denoted by H), Kroger (denoted by K), and Walmart (denoted by W) is determined by the linear functions:

$$H(x, y, z) = x + y + z$$
$$K(x, y, z) = 3x - 2y + z$$
$$W(x, y, z) = y + 4z$$

where x is the value of the companies in Canada, y is the value of the companies in the United States of America, and y is the value of the companies elsewhere.

1. During one week HEB earns \$5 million, Kroger earns \$8 million, and Walmart earns twice as much as HEB. Write a system of linear equations representing the revenues obtained for that week. *[3]*

2. Find formulas giving the equilibrium points for when the revenue of HEB is the same as the revenue of Walmart. *[2]*

1. $5 = x + y + z$
$8 = 3x - 2y + z$
$10 = y + 4z.$

2. $x + y + z = H = W = y + 4z$

So:

$x - 3z = 0$

Question 10. The Costco supply of avocados from California is of 20 tons when the market price is of \$5 million per ton, and of 4 tons when the market price is of \$1 million per ton. No avocados are supplied if the price falls below \$1 million per ton.

1. Assuming that supply is a linear function above the market price of \$1 million per ton of avocado, determine the supply equation (suggestion: treat price as the independent variable). *[4]*

2. If market equilibrium occurs at a price of \$2 million per ton of avocado, determine the market demand in tons of avocados. *[2]*

1. $S(p)$ the supply function, p the price in millions of dollars.

Assuming linearity: $S(p) = mx + ?$.

$S(1) = 4$ so $m = 4$ so $S(p) = \begin{cases} 4p, & p \geq 1 \\ 0, & p < 1 \end{cases}$.

$S(5) = 20$ $\quad ? = 0$

2. At market equilibrium, demand equals supply.

The supply is: $S(p)$ for $p = 2$, $S(2) = 8$.

Hence the demand is 8 tons of avocados.

Question 11. In August 2019, Tiff's Treats sold x dozens of cookies and y dozens of cupcakes, at a price of \$10.00 and \$8.00 per dozen respectively. The total revenue made from cookie sales was \$595.00, and the dozens of cupcakes sold was three times the dozens of cookies sold.

1. Construct a system of linear equations conveying this information. *[2]*
2. Using the method of substitution or elimination, determine the dozens of cookies and the dozens of cupcakes sold. Determine how many individual cookies and how many individual cupcakes were sold. *[4]*

1. $10x + 8y = 595$

$y = 3x$

2. $10x + 8 \cdot 3 \cdot x = 595$ so $x = \frac{595}{34} = 17.5$

so $y = 52.5$

So $17.5 \cdot 12 = 210$ cookies are sold.

$52.5 \cdot 12 = 630$ cupcakes

Question 12. Whirlpool manufactures three types of dishwashers: front load, side load, and top load. It also sells each dishwasher at a price of \$300, \$250, and \$270 respectively. The cost of manufacturing each dishwasher is of \$135, \$121, and \$78 respectively. What is the profit that Whirlpool makes from the sales of dishwashers? Construct, but do not solve, a linear programming model which can be solved to maximize the profit made, given that the following must be true: *[5]*

a) the total production output cannot exceed 230 items,

b) the number of top load dishwashers produced must be no less than five times the number of front load and side load dishwashers combined,

c) no more than 18 side load dishwashers can be manufactured.

front : f

side : s

top : t

$C(f,s,t) = 135 \cdot f + 121 \cdot s + 78 \cdot t$

$R(f,s,t) = 300 \cdot f + 250 \cdot s + 270 \cdot t$

$P(f,s,t) = R - C = 165 \cdot f + 129 \cdot s + 192 \cdot t$

Now the constraints (with the assumption that we only produce a positive amount of products) are:

$f \geq 0,\ s \geq 0,\ t \geq 0$

$f + s + t \leq 230$

$t \geq 5 \cdot (f + s)$

$s \leq 18$

objective function $P(f,s,t)$.

Question 13. Given the following system of linear equations:

$$\begin{aligned} 4x + 2y + 8z &= 16 \\ 2x + 2y + 4z &= 12 \\ 3x + 5y + 3z &= 5 \end{aligned}$$

1. Determine the augmented array representing this system. *[1]*
2. Pivot about the element in the second row and the first column. *[3]*

1.

$$\left[\begin{array}{ccc|c} 4 & 2 & 8 & 16 \\ 2 & 2 & 4 & 12 \\ 3 & 5 & 3 & 5 \end{array}\right]$$

2. We pivot a_{21}:

$$\left[\begin{array}{ccc|c} 4 & 2 & 8 & 16 \\ 2 & 2 & 4 & 12 \\ 3 & 5 & 3 & 5 \end{array}\right] \xrightarrow{R_2 \cdot \frac{1}{2}} \left[\begin{array}{ccc|c} 4 & 2 & 8 & 16 \\ 1 & 1 & 2 & 6 \\ 3 & 5 & 3 & 5 \end{array}\right] \xrightarrow{R_1 - 4 \cdot R_2} \left[\begin{array}{ccc|c} 0 & -2 & 0 & -8 \\ 1 & 1 & 2 & 6 \\ 3 & 5 & 3 & 5 \end{array}\right]$$

$$\xrightarrow{R_3 - 3 \cdot R_2} \left[\begin{array}{ccc|c} 0 & -2 & 0 & -8 \\ 1 & 1 & 2 & 6 \\ 0 & 2 & -3 & -13 \end{array}\right]$$