Math 141

Fall 2017

Exam 3A

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

Answer Key .
_____
(Signature required)

- Read all directions.
- Unless stated otherwise, answers are rounded to 4 decimal places
- **<u>BUBBLE YOUR SCANTRON COMPLETELY AND CAREFULLY</u>**
  - o MATH 141 SECTION 5** (see your label above if you don't remember)
  - o TEST FORM A
  - o IF YOU DO NOT FILL IN YOUR <u>UIN</u> AND <u>TEST FORM</u>, YOU WILL LOSE POINTS!
  - o DO NOT BUBBLE SHEET TYPE OR OPTIONS
  - o INSTRUCTOR: EPSTEIN, DATE: Fall 2017, EXAM 3A
- The scantrons will not be returned, so you MUST mark your answers on this exam.
- No partial credit on scantron questions.
- There are 3 pages with writing on both sides of every page.
- There is a 5 point deduction for any errors in following the directions so be sure your scantron is bubbled correctly.
- **NO LATE EXAMS!!** Penalty is (at least) 10 points off your score.

A standard deck of 52 cards has 13 cards in 4 different suits. The red suits are hearts and diamonds. The black suits are spades and clubs. The cards in each suit are A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. The J, Q, and K are face cards.

There are 7 days in a week and 52 weeks in a year.

There are 12 inches in a foot, 3 feet in a yard, and 5280 feet in a mile.

The English alphabet has 26 letters. There are 5 vowels (a, e, i, o and u).

Questions 1 – 25 are worth 4 points each. Mark your answers on your scantron AND your exam.

1. A cup has 3 pennies and 2 dimes. A sample of two coins is selected at random. How many outcomes are in the uniform sample space for this experiment?
   (A) 3
   (B) 4
   (C) 10
   (D) 20
   (E) None of these. Answer is _____

$C(5,2)$

2. A ring is insured for $1400 and the annual premium on the ring is $15. If there is a 0.9% chance that the ring will need to be replaced, what is the insurance company's expected gain on this policy?
   (A) $2.40
   (B) $1.35
   (C) $12.60
   (D) $13.65
   (E) None of these. Answer is _____

	replace	not replace
X	15-1400	15
P(X)	0.009	0.991

3. The odds in favor of a certain horse in a race are 2:7. What is the probability (to 2 decimal places) the horse will win the race?
   (A) 0.29
   (B) 0.22
   (C) 0.40
   (D) 0.31
   (E) None of these. Answer is _____

$\dfrac{2}{2+7}$

The next two questions use the following information:

A shelf of binders is examined and the number of dividers in each binder is counted. The table below shows the results.

# of binders	3	4	5	9
# of dividers	1 2	8	3	1

← frequency
← X

4. What is the mean?
   (A) 6
   (B) 5.25
   (C) 3.8333
   (D) 4.3810
   (E) None of these. Answer is _____

5. What is the mode?
   (A) 9 binders
   (B) 3 binders
   (C) 12 dividers
   (D) 1 divider
   (E) None of these. Answer is _____

The next three questions use the following information:

You have 12 soda bottles and each of the bottles has a 25% chance of having a prize under the cap.

6. What is the probability that exactly 3 of the sodas have a prize?
   - (A) 0.6488
   - (B) 0.7500
   - (C) 0.2581
   - (D) 0.0156
   - (E) None of these. Answer is _____

   Binomial $\quad n = 12$
   $\quad p = 0.25$
   $\quad x = 3$

7. What is the probability that more than 6 sodas have a prize?
   - (A) 0.9456
   - (B) 0.9857
   - (C) 0.0544
   - (D) 0.0143
   - (E) None of these. Answer is _____

   Binomial as above $\quad x = 7, ..., 12.$

8. What is the probability that exactly 2 of the first 6 sodas have a prize and exactly 1 of the last 6 sodas have a prize?
   - (A) 0.6526
   - (B) 0.1056
   - (C) 0.1111
   - (D) 0.1667
   - (E) None of these. Answer is _____

   $$\left(C(6,2)\, p^x q^{n-x} \right) \cdot \left(C(6,1)\, p^x q^{n-x} \right)$$
   $n=6, x=2, p=0.25 \qquad n=6, x=1, p=0.25$

   (Binomial as above)

9. You are dealt a hand of 5 cards from a well-shuffled standard deck of 52 cards. What is the probability you have at least one ace in the hand?
   - (A) 0.6588
   - (B) 0.7005
   - (C) 0.2995
   - (D) 0.3412
   - (E) None of these. Answer is _____

   $$P(\text{at least one ace}) = 1 - P(\text{no ace}) = 1 - \frac{C(4,0) \cdot C(48,5)}{C(52,5)}$$

   $$P(E) = \frac{n(E)}{n(S)}.$$

10. There are 32 runners in a race. Seven runners take more than 8 minutes to run the race, 15 runners take 6 minutes or more to run the race and 5 runners take 4 minutes or less to run the race. What is the probability that a runner takes between 4 and 6 minutes to run the race?
    - (A) 0.3750
    - (B) 0.1563
    - (C) 0.6000
    - (D) 0.4444
    - (E) None of these. Answer is _____

$\bar{x} \le 4$	$4 < \bar{x} < 6$	$6 \le \bar{x} \le 8$	$\bar{x} > 8$	
5	12	8	7	frequency
5/32	12/32	8/32	7/32	probability

The next two questions use the data in the table below:

X	1	2	7	11	20
Frequency	11	6	5	2	1

11. What is the median?
   (A) 2
   (B) 7
   (C) 11
   (D) 5
   (E) None of these. Answer is _____

12. What is the standard deviation (to two decimal places)?
   (A) 4.57
   (B) 2.14
   (C) 3.52
   (D) 6.91
   (E) None of these. Answer is _____

13. A bag of apples is purchased and each of the apples are weighed. The heaviest apple weighs less than 110 grams and the lightest weighs more than 45 grams. What type of random variable is w, the weight of an apple in grams? What are the possible values of w?
   (A) continuous with $45 < w < 110$
   (B) continuous with $45 \le w \le 110$
   (C) finite discrete with $w = 45, 46, \ldots 110$
   (D) finite discrete with $w = 46, 47, \ldots 109$
   (E) None of these. Answer is _____

The next two questions refer to the following experiment:

A drawer has 6 forks, 4 knives and 9 spoons. Five items are chosen at random from this drawer.

14. What is the probability you choose exactly two forks and exactly two spoons?
   (A) 0
   (B) 0.1393
   (C) 0.0464
   (D) 0.1858
   (E) None of these. Answer is _____

$$\frac{C(6,2) \cdot C(9,2) \cdot C(4,1)}{C(19,5)} = \frac{2160}{11628}$$

15. What is the probability you choose exactly two forks or exactly two spoons?
   (A) 0.6940
   (B) 0.7405
   (C) 0.5547
   (D) 0.5805
   (E) None of these. Answer is _____

$$\frac{C(6,2) \cdot C(13,3) + C(9,2) \cdot C(10,3) - C(6,2) \cdot C(9,2) \cdot C(4,1)}{C(19,5)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

16. A store has 90 printers in stock. It is known that 40 printers have scanners, 70 have wireless or a scanner, and 60 have wireless. What is the probability that a wireless printer has a scanner?

(A) 0.3
(B) 0.5
(C) 0.6
(D) 0.7
(E) None of these. Answer is _____

$$P(S|W) = \frac{n(S \cap W)}{n(W)} = \frac{30}{60}$$

$$n(S \cup W) = n(S) + n(W) - n(W \cap S) \quad \text{so} \quad n(W \cap S) = 30.$$

17. A family has 3 cordless phones. The independent probabilities that each phone is not in its cradle are 25%, 30%, and 10%. What is the probability that exactly one phone is not in its cradle?

(A) 0.0075
(B) 0.6500
(C) 0.4125
(D) 0.4725
(E) None of these. Answer is _____

$$\underbrace{0.25}_{\text{not}} \cdot \underbrace{0.7}_{\text{in}} \cdot \underbrace{0.9}_{\text{in}} + \underbrace{0.75}_{\text{in}} \cdot \underbrace{0.3}_{\text{not in}} \cdot \underbrace{0.9}_{\text{in}} + \underbrace{0.75}_{\text{in}} \cdot \underbrace{0.7}_{\text{in}} \cdot \underbrace{0.1}_{\text{not}}$$

$$\frac{1st\,p.\quad 2nd\,p.\quad 3rd\,p.}{}$$

18. Two cards are drawn in succession without replacement from a standard well-shuffled deck of 52 cards. What is the probability that the first card drawn was a heart given that the second card drawn was a club?

(A) 13/51
(B) 13/52
(C) 12/51
(D) 12/52
(E) None of these. Answer is _____
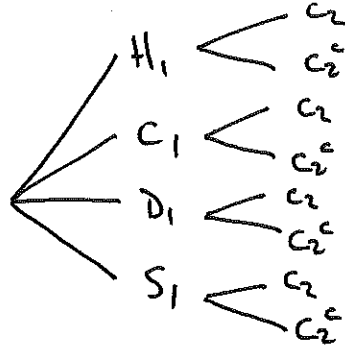
$$P(H_1|C_2) = \frac{\frac{13}{52} \cdot \frac{13}{51}}{4 \cdot \frac{13}{52} \cdot \frac{13}{51}}$$

The next two questions use the information below:

The table shows the demographics of a college for each age group how many are enrolled in undergraduate and graduate studies:

age groups:	17 – 21	22 – 25	26 – 30	31 and older
Percent in the age group	52%	36%	8%	4%
Percent of age group that are undergraduates	98%	55%	39%	20%

19. What is the probability that a randomly selected student is an undergraduate?

(A) 0.5096
(B) 0.7468
(C) 0.5300
(D) 1
(E) None of these. Answer is _____

$$P(u) = 0.98 \cdot 0.52 + 0.36 \cdot 0.55 + 0.08 \cdot 0.39 + 0.04 \cdot 0.2$$

**20.** What is the probability that an undergraduate is in the 22 – 25 age group?

(A) 0.4821
(B) 0.2651
(C) 0.5500
(D) 0.7365

$$P(22\text{-}25|u) = \frac{P(\{22\text{-}25\} \cap u)}{P(u)}$$

(E) None of these. Answer is _____

21. A medical test has been developed to detect xyzzy disease. It is estimated that 5% of the patients who come in for the test have the disease. When the test is given to a patient who has xyzzy disease, it is detected (positive) 90% of the time. When given to a patient who does not have xyzzy disease, a positive result is returned 15% of the time. What is the probability that a person who tests positive does not have xyzzy disease?

    (A) 0.1425
    (B) 0.7600
    (C) 0.8000
    (D) 0.1500
    (E) None of these. Answer is _____
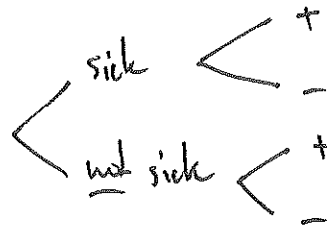
$$P(\text{not sick} \mid +) = \frac{0.95 \cdot 0.15}{0.05 \cdot 0.9 + 0.95 \cdot 0.15}$$

22. An experiment with sample space $S = \{s_1, s_2, s_3, s_4, s_5\}$ has the probability distribution given below. Given that $A = \{s_2, s_3, s_5\}$, that s_3 and s_4 are equally likely, and that $P(A^c) = 0.42$, find p_5.

Simple Event	$\{s_1\}$	$\{s_2\}$	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$
Probability	0.14	0.17	p_3	p_4	p_5

    (A) Not enough information given
    (B) 0.13
    (C) 0.23
    (D) 0.28
    (E) None of these. Answer is _____

$$P(A) = 1 - P(A^c) = 0.58 \qquad p_3$$
$$0.42 = P(A^c) = 0.14 + p_4 \quad \text{so} \quad p_4 = 0.28$$
$$0.58 = P(A) = 0.17 + 0.28 + p_5 \quad \text{so} \quad p_5 = 0.13$$

23. A certain medicine has two main side effects, headaches and sleepiness. The chance a patient has headaches is 30%, the chance of sleepiness is 25% and the chance of headaches or sleepiness is 47.5%. Which statement below is true

    (A) Headaches and sleepiness are independent of each other
    (B) Headaches and sleepiness are not independent of each other
    (C) There is not enough information to determine if headaches and sleepiness are independent
    (D) Headaches and sleepiness are mutually exclusive
    (E) None of these. Answer is _____

$$P(H \cup S) = P(H) + P(S) - P(H \cap S) \quad \text{so} \quad P(H \cap S) = 0.75.$$
$$P(H) \cdot P(S) = 0.75$$

24. What does the standard deviation measure?

    (A) On average, how far the data is from the median
    (B) On average, how far the data is from the mean
    (C) On average, how annoying the data is
    (D) On average, how accurate the data is
    (E) None of these. Answer is _____

25. Use the histogram to the right to determine the mode or modes of the data:

    (A) c and d
    (B) a
    (C) b and e
    (D) no mode
    (E) None of these. Answer is _____