Name and NetID:

Question 1. Using either the method of substitution or elimination, determine the value of a for which the following system of equations has no solution: *[4]*

$$2x - y = 10, \quad ax + 7y = 5.$$

Solution. Using the method of substitution, from the first equation we obtain that:

$$y = 2x - 10$$

and substituting this in the second equation yields:

$$ax + 7(2x - 10) = 5 \quad \text{so} \quad ax + 14x - 70 = 0 \quad \textbf{so} \quad (a + 14)x = 70.$$

We know that there is no value of x such that $0 = 0 \cdot x = 70$, so if $a + 14 = 0$ this last equation has no solution. Thus for $\boxed{a = -14}$ the system of equations has no solution.

Question 2. Use Gauss-Jordan elimination to solve the system of linear equations: *[6]*

$$x + y + 4z = 31, \quad 2x + 2y - z = 8, \quad 3x + 3y - 3z = 3.$$

Solution. We first form the augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 4 & 31 \\ 2 & 2 & -1 & 8 \\ 3 & 3 & -3 & 3 \end{array}\right]$$

and then we do Gauss-Jordan elimination:

$$\left[\begin{array}{ccc|c} 1 & 1 & 4 & 31 \\ 2 & 2 & -1 & 8 \\ 3 & 3 & -3 & 3 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 4 & 31 \\ 0 & 0 & -9 & -54 \\ 3 & 3 & -3 & 3 \end{array}\right] \xrightarrow{R_3 - 3R_1} \left[\begin{array}{ccc|c} 1 & 1 & 4 & 31 \\ 0 & 0 & -9 & -54 \\ 0 & 0 & -15 & -90 \end{array}\right] \xrightarrow{\frac{1}{-9}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 4 & 31 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & -15 & -90 \end{array}\right] \xrightarrow{\frac{1}{-15}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 4 & 31 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 1 & 6 \end{array}\right] \xrightarrow{R_1 - 4R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 7 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 1 & 6 \end{array}\right] \xrightarrow{R_3 - R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 7 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

which translates into $z = 6$, y is a free variable, and $x + y = 7$ so $x = 7 - y$. Setting $t \in \mathbb{R}$ a parameter, the general solution is $\boxed{x = 7 - t, \quad y = t, \quad z = 6}$.

Bonus Question. Use Gauss-Jordan elimination to solve the system of linear equations: *[4]*

$$2x + 3y - 2z = 10, \quad 3x - 2y + 2z = 0, \quad 4x - y + 3z = -1.$$

Solution. We first form the augmented matrix:

$$\left[\begin{array}{ccc|c} 2 & 3 & -2 & 10 \\ 3 & -2 & 2 & 0 \\ 4 & -1 & 3 & -1 \end{array}\right]$$

and then we do Gauss-Jordan elimination:

$$\left[\begin{array}{ccc|c} 2 & 3 & -2 & 10 \\ 3 & -2 & 2 & 0 \\ 4 & -1 & 3 & -1 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 3/2 & -1 & 5 \\ 3 & -2 & 2 & 0 \\ 4 & -1 & 3 & -1 \end{array}\right] \xrightarrow{R_2-3R_1} \left[\begin{array}{ccc|c} 1 & 3/2 & -1 & 5 \\ 0 & -13/2 & 5 & 15 \\ 4 & -1 & 3 & -1 \end{array}\right] \xrightarrow{R_3-4R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 3/2 & -1 & 5 \\ 0 & -13/2 & 5 & 15 \\ 0 & -7 & 7 & -21 \end{array}\right] \xrightarrow{R_2\leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 3/2 & -1 & 5 \\ 0 & -7 & 7 & -21 \\ 0 & -13/2 & 5 & 15 \end{array}\right] \xrightarrow{\frac{1}{-7}R_2} \left[\begin{array}{ccc|c} 1 & 3/2 & -1 & 5 \\ 0 & 1 & -1 & 3 \\ 0 & -13/2 & 5 & 15 \end{array}\right] \xrightarrow{R_1-\frac{3}{2}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & -1 & 3 \\ 0 & -13/2 & 5 & 15 \end{array}\right] \xrightarrow{R_3+13R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & -3 & 9 \end{array}\right] \xrightarrow{\frac{1}{-3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -3 \end{array}\right] \xrightarrow{R_1-\frac{1}{2}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -3 \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

which translates into $\boxed{x = 2, \quad y = 0, \quad z = -3}$, which is the desired solution.