NAME AND NETID:

Question 1. Compute the matrix $(2AB - 3C)^T$, given the following: *[4]*

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & x & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 2 & 0 \\ 3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}.$$

Solution. We have:

$$AB = \begin{bmatrix} 7 & 4 \\ 2x & 1 \end{bmatrix} \quad \text{so} \quad 2AB = \begin{bmatrix} 14 & 8 \\ 4x & 2 \end{bmatrix}$$

also:

$$3C = \begin{bmatrix} 3 & 6 \\ 12 & 9 \end{bmatrix} \text{so} \quad 2AB - 3C = \begin{bmatrix} 11 & 2 \\ 4x - 12 & -7 \end{bmatrix}$$

so finally:

$$\boxed{(2AB - 3C)^T = \begin{bmatrix} 11 & 4x - 12 \\ 2 & -7 \end{bmatrix}}.$$

Question 2. A supermarket sells ice cream cones of three different flavors: chocolate, strawberry, and vanilla. Due to different membership cards, Alice, Bob, and Charlie pay according to the following table:

	Chocolate	Strawberry	Vanilla
Alice	\$2.00	\$2.50	\$2.00
Bob	\$1.00	\$3.00	\$1.00
Charlie	\$1.00	\$1.50	\$2.00

assuming that everyone purchases x chocolate, y strawberry, and z vanilla cones:

1. Write a system of linear equations representing the amounts spent by each. *[2]*
2. Convert the above system into matrix format, and assuming $x = 3$, $y = 8$, $z = 5$, determine the total amount spent. *[4]*

Solution. Each spends the amount of cones multiplied by the cost per cone. Setting A, B, and C the amounts spent by Alice, Bob, and Charlie respectively, the system of linear equations is:

$$\boxed{\begin{aligned} 2x + 2.5y + 2z &= A \\ 1x + 3y + 1z &= B \\ 1x + 1.5y + 2z &= C \end{aligned}}.$$

which has matrix form:

$$\boxed{\left[\begin{array}{ccc|c} 2 & 2.5 & 2 & -A \\ 1 & 3 & 1 & -B \\ 1 & 1.5 & 2 & -C \end{array}\right]} \quad \text{or} \quad \boxed{\left[\begin{array}{ccc|c} -2 & -2.5 & -2 & A \\ -1 & -3 & -1 & B \\ -1 & -1.5 & -2 & C \end{array}\right]}$$

and setting $x = 3$, $y = 8$, $z = 5$ we have:

$$\begin{aligned} A &= 2 \cdot 3 + 2.5 \cdot 8 + 2 \cdot 5 = 36 \\ B &= 1 \cdot 3 + 3 \cdot 8 + 1 \cdot 5 = 32 \\ C &= 1 \cdot 3 + 1.5 \cdot 8 + 2 \cdot 5 = 25 \end{aligned}$$

so the total amount spent is $\boxed{A + B + C = \$93}$.

Bonus Question. A supermarket sells ice cream cones of three different flavors: chocolate, strawberry, and vanilla. Due to different membership cards, Alice, Bob, and Charlie pay according to the following table:

	Chocolate	Strawberry	Vanilla
Alice	\$2.00	\$2.50	\$2.00
Bob	\$1.00	\$3.00	\$1.00
Charlie	\$1.00	\$1.50	\$2.00

assuming that everyone purchases x chocolate, y strawberry, and z vanilla cones, write the equations describing the linear programming problem which arises if we wish to maximize the total amount spent, subject to the constraints: at least one cone of each flavor must be bought, the total number of cones purchased cannot exceed thirty, and the number of chocolate and strawberry cones bought must together be greater than twice the number of vanilla cones bought. *[4]*

Solution. Each spends the amount of cones multiplied by the cost per cone. Setting A, B, and C the amounts spent by Alice, Bob, and Charlie respectively, the equations describing the amount spent by each are:

$$\begin{aligned} A &= 2x + 2.5y + 2z \\ B &= 1x + 3y + 1z \\ C &= 1x + 1.5y + 2z \end{aligned}$$

so the total amount spent is $\boxed{T = A + B + C = 4x + 7y + 5z}$, this is the objective function to maximize. The constraints are:

1. at least one cone of each flavor must be bought: $\boxed{x \geq 1}$, $\boxed{y \geq 1}$, $\boxed{z \geq 1}$,
2. the total number of cones purchased cannot exceed thirty: $\boxed{x + y + z \leq 30}$,
3. the number of chocolate and strawberry cones bought must together be greater than twice the number of vanilla cones bought: $\boxed{x + y > 2z}$.