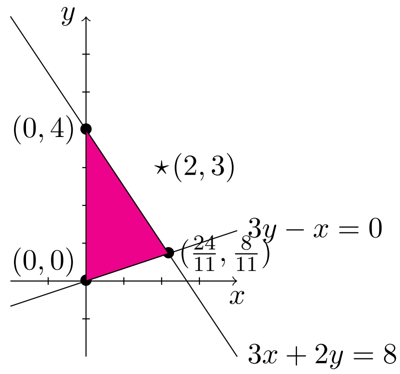
Name and NetID:

Question 1. Provide a detailed graphical representation of the solution set to the following system of inequalities, and determine whether $(2, 3)$ lies within. *[4]*

$$3x + 2y \leq 8, \quad x > 0, \quad y > 0, \quad 3y - x \geq 0.$$

Solution. The graphical representation with the intersection points is:



without the segment (equivalently, with a dotted line) from $(0, 0)$ to $(0, 4)$. We can see in the picture that $\boxed{(2, 3) \text{ does not lie within the solution set}}$. This can be justified algebraically since $3 \cdot 2 + 2 \cdot 3 = 12 \not\leq 8$ so $(2, 3)$ does not satisfy $3x + 2y \leq 8$.

Question 2. A supermarket sells chocolate and strawberry ice cream cones. Due to different membership cards, Alice and Bob pay according to the following table:

	Chocolate	Strawberry
Alice	\$2.75	\$2.00
Bob	\$1.25	\$2.00

assuming that both purchase x chocolate and y strawberry cones, solve for x and y the linear programming problem which arises if we wish to maximize the total amount spent, subject to the constraints: at least one cone of each flavor must be bought, the total number of cones purchased cannot exceed fifteen, and the number of chocolate cones bought must be at least as many as the number of strawberry cones bought. *[6]*

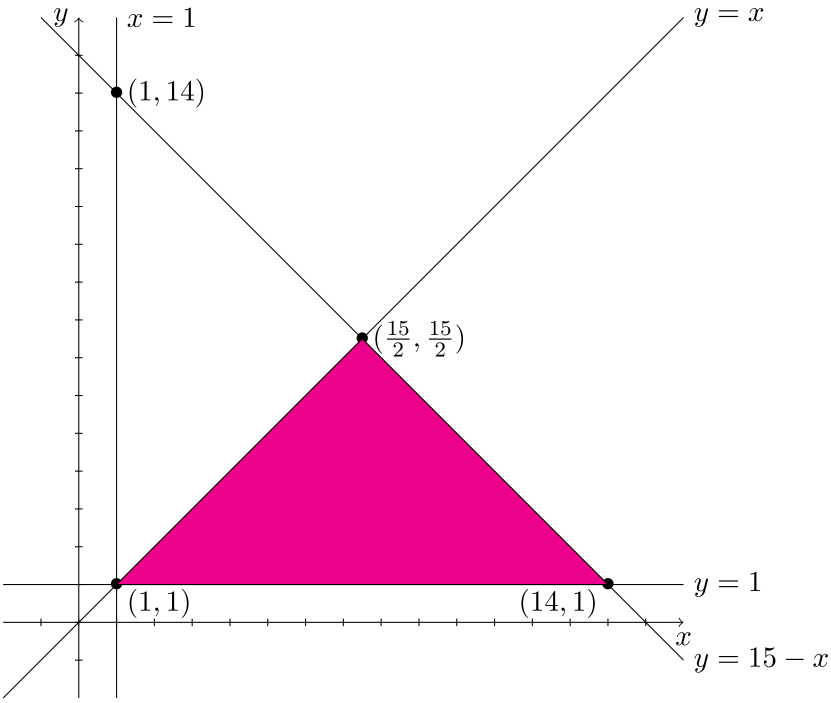
Solution. Each spends the amount of cones multiplied by the cost per cone. Setting A and B the amounts spent by Alice and Bob respectively, the equations describing the amount spent by each are:

$$A = 2.75x + 2y$$
$$B = 1.25x + 2y$$

so the total amount spent is $\boxed{T = A + B = 4x + 4y}$, this is the objective function to maximize. The constraints are:

1. at least one cone of each flavor must be bought: $\boxed{x \geq 1}$, $\boxed{y \geq 1}$,
2. the total number of cones purchased cannot exceed fifteen: $\boxed{x + y \leq 15}$,
3. the number of chocolate cones bought must be at least as many as the number of strawberry cones bought: $\boxed{x \geq y}$.

The graphical representation with the intersection points is:



and when we evaluate the objective function on the corners of the region we obtain:

$$T(1,1) = 4 \cdot 1 + 4 \cdot 1 = 8$$
$$T(7.5, 7.5) = 4 \cdot 1 + 4 \cdot 11 = 60$$
$$T(14, 1) = 4 \cdot 6 + 4 \cdot 6 = 60$$

so since the maximum is attained at the two corners $(7.5, 7.5)$ and $(14, 1)$, the whole segment between $(7.5, 7.5)$ and $(14, 1)$ maximizes the objective function. Since x and y represent the amount of ice cream cones sold, they can only be natural numbers, so the objective function is maximized at the points:

$$\boxed{(8,7), (9,6), (10,5), (11,4), (12,3), (13,2), (14,1)}.$$