Name and NetID:

Question 1. Three cards are drawn from a deck having three suits: red, green, and blue. Let $\mathcal{S}$ be the sample space of the suits of all drawn hands of three cards, A be the event where the final card is green, and B be the event where the first card is red.

1. Calculate, providing reasoning, the size of $\mathcal{S}$. *[3]*
2. List the elements of $A \cap B$ and the elements of $(A \cap B)^c$. *[3]*

Solution.

1. Since in the second part of the problem we are asked about order, the sample space should take order into account. We then want the number of distinct hands of three cards where order matters. Assuming that we do not run out of cards (that is, we have at least three cards of each suite), we have three options (namely, each of the three suits) for each of the three positions, so then:

$$\boxed{n(\mathcal{S}) = 3 \cdot 3 \cdot 3 = 27}.$$

2. The event $A \cap B$ is formed by the hands where the first card is red (because of B) and where the last card is green (because of A). We then have three options (namely, each of the three suits) for the second card:

$$\boxed{A \cap B = \{RRG, RGG, RBG\}}.$$

Then the complement is formed by the remaining hands:

$$\begin{aligned}(A \cap B)^c = \{&RRR, RRB, RGR, RGB, RBR, RBB,\\ &GRR, GRG, GRB, GGR, GGG, GGB, GBR, GBG, GBB,\\ &BRR, BRG, BRB, BGR, BGG, BGB, BBR, BBG, BBB\}.\end{aligned}$$

Question 2. The Texas A&M Student Body Senate is electing committee members from a pool of twelve candidates, intending to choose five persons. Three candidates come from the College of Science, three from the College of Medicine, four from the College of Education, and two from the College of Engineering. Determine the total number of distinct committees possible, given that at least one person must be chosen from each College. *[4]*

Solution. Consider each of the four departments as contributing with candidates to a slot in a bracket: [Science, Medicine, Education, Engineering]. Since we have that the committee must be of five members, all the slots in the brackets must add up to five. We then have the following four possible ways of choosing the candidates:

$$[2,1,1,1], \quad [1,2,1,1] \quad ,[1,1,2,1], \quad [1,1,1,2].$$

Since ordering doesn't matter, it follows that the total number of distinct committees for each of the four options above is:

$$\begin{array}{lll} [2,1,1,1] & \text{can be chosen as} & C(3,2)\cdot C(3,1)\cdot C(4,1)\cdot C(2,1)=72 \\ [1,2,1,1] & \text{can be chosen as} & C(3,1)\cdot C(3,2)\cdot C(4,1)\cdot C(2,1)=72 \\ [1,1,2,1] & \text{can be chosen as} & C(3,1)\cdot C(3,1)\cdot C(4,2)\cdot C(2,1)=108 \\ [1,1,1,2] & \text{can be chosen as} & C(3,1)\cdot C(3,1)\cdot C(4,1)\cdot C(2,2)=36 \end{array}$$

and thus the total number of distinct committees possible is:

$$72+72+108+36=\boxed{288}.$$

Bonus Question.

1. Consider subsets A and B of a universal set U. Describe in your own words the sets $A^c\cup B^c$ and $(A\cup B)^c$. *[2]*

2. Consider subsets A, B and C of a universal set U, satisfying $n(A^c\cup B\cup C^c)=27$ and $n(U)=40$. Explicitly use De Morgan's laws to compute $n(A\cap B^c\cap C)$. *[2]*

Solution.

1. $A^c\cup B^c=(A\cap B)^c$ is the set formed by the elements that do not belong to both A and B (simultaneously). $(A\cup B)^c=A^c\cap B^c$ is the set formed by the elements that are not in A and not in B.

2. We can use De Mogan's Laws twice:

$$\begin{aligned}(A^c\cup B\cup C^c)^c &= ((A^c\cup B)\cup C^c)^c=(A^c\cup B)^c\cap (C^c)^c \\ &= (A^c\cup B)^c\cap C=(A^c)^c\cap B^c\cap C=A\cap B^c\cap C\end{aligned}$$

and now, keeping in mind that the size of a set plus the size of its complement is the size of the universal set:

$$\boxed{n(A\cap B^c\cap C)=n((A^c\cup B\cup C^c)^c)=n(U)-n(A^c\cup B\cup C^c)=13}.$$