NAME AND NETID:

Question 1. The probability of a Texas A&M student of going to a football game depends on how long they have been at the University and what is their favorite color. Let F represent a freshman, S a sophomore, and J a junior, while M if their favorite color is maroon. The probabilities are:

$$P(F) = 0.23, \quad P(S) = 0.30, \quad P(J) = 0.17, \quad P(M) = 0.20, \quad P(S \cap M) = 0.08.$$

1. Calculate the probability that a student going to a game is a sophomore or has maroon as their favorite color. *[2]*
2. Calculate the probability that a student going to a game does not have maroon as their favorite color. *[2]*
3. Using the fact that being in a different year describes mutually exclusive events, bound the probability that a student going to a game is a senior. *[2]*

Solution.

1. Using the counting principle in the context of probability we obtain:

$$\boxed{P(S \cup M) = P(S) + P(M) - P(S \cap M) = 0.30 + 0.20 - 0.08 = 0.42}.$$

2. Recall that the probability of an event happening and the probability of their complement happening has to add up to one, so:

$$\boxed{P(M^c) = 1 - P(M) = 1 - 0.2 = 0.8}.$$

3. The students can either be freshmen, sophomores, juniors, seniors (an event that we denote by X), or they may have been in University for five or more years (an event that we denote by Y). Since these are mutually exclusive events, they must add up to one, and thus:

$$1 = P(F)+P(S)+P(J)+P(X)+P(Y) \quad \text{so} \quad P(X) = 1-P(F)-P(S)-P(J)-P(Y)$$

and since these are probabilities, all of them are equal or bigger than zero, so in particular $P(Y) \geq 0$ and we obtain:

$$\begin{aligned} P(X) &= 1 - P(F) - P(S) - P(J) - P(Y) \\ &\leq 1 - P(F) - P(S) - P(J) \\ &= 1 - 0.23 - 0.30 - 0.17 = 0.30 \quad \text{so} \quad \boxed{P(X) \leq 0.30}. \end{aligned}$$

Question 2. Three cards are drawn from a deck of thirty playing cards separated into three equally portioned suits: red, green, and blue. Let $\mathcal{S}$ be the sample space of the suits of all drawn hands of three cards.

1. Assuming cards are drawn without replacement, calculate the probability that a blue is drawn first and a green last. *[2]*

2. Assuming cards are drawn with replacement, calculate the probability of at least two reds being drawn. *[2]*

Solution.

1. The hands where a blue card is drawn first and a green last are $\{BRG, BGG, BBG\}$. We can then directly count the probability of each of these hands happening, and then add them up:

$$\begin{aligned} P(\{BRG, BGG, BBG\}) &= P(BRG) + P(BGG) + P(BBG) \\ &= \frac{10}{30} \cdot \frac{10}{29} \cdot \frac{10}{28} + \frac{10}{30} \cdot \frac{10}{29} \cdot \frac{9}{28} + \frac{10}{30} \cdot \frac{9}{29} \cdot \frac{10}{28} \\ &= \frac{10}{87} \quad \text{so} \quad \boxed{P(\{BRG, BGG, BBG\}) = \frac{10}{87}}. \end{aligned}$$

2. For the condition to be satisfied, our hand has to either have exactly two red cards, or all three red cards. The probability of getting a hand with all three cards red is $P(RRR) = (10/30) \cdot (10/30) \cdot (10/30)$. If we have only two cards, then these are either in the first two positions (and the third position is green or blue, denote it by X), which happens with probability $P(RRX) = (10/30) \cdot (10/30) \cdot (20/30)$, or the two cards are in the first and third positions (and the second position is green or blue, denote it by X), which happens with probability $P(RXR) = (10/30) \cdot (20/30) \cdot (10/30)$, or the two cards are in the last two positions (and the first position is green or blue, denote it by X), which happens with probability $P(XRR) = (20/30) \cdot (10/30) \cdot (10/30)$. Adding everything up, we obtain:

$$\boxed{P(n(R) \geq 2) = \frac{10}{30} \cdot \frac{10}{30} \cdot \frac{10}{30} + \frac{10}{30} \cdot \frac{10}{30} \cdot \frac{20}{30} + \frac{10}{30} \cdot \frac{20}{30} \cdot \frac{10}{30} + \frac{20}{30} \cdot \frac{10}{30} \cdot \frac{10}{30} = \frac{7}{27}}.$$

Bonus Question. The New England Patriots have a stock of 100 game balls. An inspection into this stock has yielded that at least 45 of the balls are deflated per NFL standards. If Tom Brady randomly selects from the stock eight balls for the game, determine the probability:

1. That three of them are deflated. *[2]*

2. That at least one of them is deflated. *[2]*

Solution.

1. The event E that we are interested in is randomly choosing 3 deflated balls out of 45, and this can be done in $C(45,3)$ ways, and then choosing 5 non deflated balls out of 55, and this can be done in $C(55,5)$. The total possible number of ways in which we can randomly select 8 balls from 100 is $C(100,8)$, and thus the probability of E happening is:

$$\boxed{P(E) = \frac{n(E)}{n(S)} = \frac{C(45,3)C(55,5)}{C(100,8)} = \frac{49862241}{187967570} \approx 0.2653}.$$

2. The event E of choosing at least one deflated ball has as complement the event E^c of choosing all of the balls to be deflated, that is, randomly choosing 8 balls out of 45. The probability of E happening is thus:

$$\boxed{P(E) = 1 - P(E^c) = 1 - \frac{n(E^c)}{n(S)} = \frac{C(45,8)}{C(100,8)} = \frac{19763141}{19786060} \approx 0.9988}.$$