NAME AND NETID:

Question 1. Let A_1, A_2 and A_3 be mutually exclusive and exhaustive events of some sample space $\mathcal{S}$, where $P(A_1) = P(A_2) = 0.3$. Suppose that one out of ten of A_1 observations support Darwin's Theory of Evolution, one out of two of A_2 observations support Darwin's Theory of Evolution, and two out of five of A_3 observations support Darwin's Theory of Evolution. Given that a certain observation does not support Darwin's Theory of Evolution, calculate the probability that it is classified in A_3. *[5]*

Solution. Since the events A_1, A_2 and A_3 are mutually exclusive and exhaustive, we have that their probabilities must add up to one, and thus $P(A_3) = 1 - P(A_1) - P(A_2) = 0.4$. We call B the event that an observation supports Darwin's Theory of Evolution, we want to compute $P(A_3|B^c)$. To compute it, we use Bayes' Theorem:

$$\begin{aligned} P(A_3|B^c) &= \frac{P(A_3)P(B^c|A_3)}{P(A_1)P(B^c|A_1) + P(A_2)P(B^c|A_2) + P(A_3)P(B^c|A_3)} \\ &= \frac{P(A_3)(1 - P(B|A_3))}{P(A_1)(1 - P(B|A_1)) + P(A_2)(1 - P(B|A_2)) + P(A_3)(1 - P(B|A_3))} \\ &= \frac{0.4 \cdot (1 - 2/5)}{0.3 \cdot (1 - 1/10) + 0.3 \cdot (1 - 1/2) + 0.4 \cdot (1 - 2/5)} = \frac{4}{11} \end{aligned}$$

and thus $\boxed{P(A_3|B^c) = 4/11}$.

Question 2. The probability of a Texas A&M student of going to a football game depends on how long they have been at the University and what is their favorite color. Let F represent a freshman, S a sophomore, and J a junior, while M if their favorite color is maroon. The probabilities are:

$$P(F) = 0.42, \quad P(S) = 0.21, \quad P(J) = 0.12, \quad P(M) = 0.25.$$

1. Given that $P(S \cap M) = 0.11$, calculate the probability that a student going to a game has maroon as their favorite color given it is a sophomore. *[3]*
2. Assuming that having maroon as a favorite color is independent of how long a student has been at the University, calculate the probability that a student going to a game has maroon as their favorite color given it is a sophomore. *[2]*

Solution.

1. We use the usual formula for conditional probability to establish:

$$\boxed{P(M|S) = \frac{P(M \cap S)}{P(S)} = \frac{0.11}{0.21} = \frac{11}{21}}.$$

2. If having maroon as a favorite color is independent of how long a student has been at the University, we then have that the conditional probability does not depend on being a sophomore, that is, $\boxed{P(M|S) = P(M) = 0.25}$.

Bonus Question. Let $\mathcal{S} = \{0, 1, 2, 4, 5, 6, 7, 8\}$ be a uniform sample space. Determine the probability that a chosen number is greater than 5 given that it is even. *[4]*

Solution. We call F the event of the chosen number being greater than 5, and we call E the event of the chosen number being even. Notice that the probability of a chosen number being even is $P(E) = n(E)/n(S) = 5/8$, and the probability of a number being bigger than five and even is $P(F \cap E) = n(E \cap F)/n(S) = 2/8$. We can now use the usual formula for conditional probability to establish:

$$\boxed{P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{2/8}{5/8} = \frac{2}{5}}.$$