Name and NetID:

Question 1. Let X be a random variable, and answer the following questions based on the probability distribution table below.

x	0	1	3	4	5	7	8	10
$P(X = x)$	0.15	0.10	0.15	0.20	0.20	0.05	0.05	0.10

1. Calculate $P(4 \le X \le 7)$. *[1]*
2. Calculate the expected value $E(X)$. *[2]*
3. Calculate the standard deviation σ_X. *[3]*

Solution.

1. We have:

$$P(4 \le X \le 7) = P(X = 4) + P(X = 5) + P(X = 7) = 0.20 + 0.20 + 0.05 = \boxed{0.45}.$$

2. We have:

$$E(X) = 0\cdot 0.15+1\cdot 0.10+3\cdot 0.15+4\cdot 0.20+5\cdot 0.20+7\cdot 0.05+8\cdot 0.05+10\cdot 0.10 = \boxed{4.1}.$$

3. We have:

$$\begin{aligned}\mathrm{Var}(X) &= 0.15 \cdot (0 - 4.1)^2 + 0.10 \cdot (1 - 4.1)^2 + 0.15 \cdot (3 - 4.1)^2 \\ &\quad + 0.20 \cdot (4 - 4.1)^2 + 0.20 \cdot (5 - 4.1)^2 + 0.05 \cdot (7 - 4.1)^2 \\ &\quad + 0.05 \cdot (8 - 4.1)^2 + 0.10 \cdot (10 - 4.1)^2 = 8.49.\end{aligned}$$

so $\boxed{\sigma_X = \sqrt{\mathrm{Var}(X)} = \sqrt{8.49} \approx 2.91376}$.

Question 2. Let X be a binomial random variable following a binomial distribution. It measures the number of days a factory has an accident in 2023. If the expected value is 5.12, then determine the probability that there is an accident any given day. *[1]*

Solution. We have $n = 365$ for the days in a year, and since $5.12 = E(X) = np$ we have that the probability of an accident any particular day is $\boxed{p = 128/9125 \approx 0.014}$.

Question 3. A doll factory takes a sample of thirteen dolls and records the masses. Determine the median and sample standard deviation of the doll's masses: *[3]*

$8.0kg$, $7.8kg$, $7.9kg$, $8.0kg$, $8.1kg$, $8.0kg$, $7.8kg$, $8.1kg$, $7.9kg$, $8.0kg$, $7.8kg$, $8.0kg$, $7.9kg$.

Solution. We can arrange the dolls' masses in increasing order:

$$7.8,\ 7.8,\ 7.8,\ 7.9,\ 7.9,\ 7.9,\ 8.0,\ 8.0,\ 8.0,\ 8.0,\ 8.0,\ 8.1,\ 8.1,$$

and thus the median is $\boxed{8.0kg}$. Moreover, we have:

$$\overline{x} = \frac{8.0+7.8+7.9+8.0+8.1+8.0+7.8+8.1+7.9+8.0+7.8+8.0+7.9}{13} = \frac{1033}{130}$$

so $\overline{x} \approx 7.95$. Thus:

$$\sigma^2 = \frac{1}{13}\cdot\Bigg(\left(8.0-\frac{1033}{130}\right)^2+\left(7.8-\frac{1033}{130}\right)^2+\left(7.9-\frac{1033}{130}\right)^2 + \left(8.0-\frac{1033}{130}\right)^2+\left(8.1-\frac{1033}{130}\right)^2+\left(8.0-\frac{1033}{130}\right)^2 + \left(7.8-\frac{1033}{130}\right)^2+\left(8.1-\frac{1033}{130}\right)^2+\left(7.9-\frac{1033}{130}\right)^2 + \left(8.0-\frac{1033}{130}\right)^2+\left(7.8-\frac{1033}{130}\right)^2+\left(8.0-\frac{1033}{130}\right)^2 + \left(7.9-\frac{1033}{130}\right)^2\Bigg) \quad \text{so} \quad \boxed{\sigma \approx 0.10088}.$$

Bonus Question. Suppose X is a normal random variable with $\mu = 50$ and $\sigma = 15$. Find the values of $\mathbb{P}(X < 55)$, $P(X > 45)$, and $P(45 < X < 55)$. *[4]*

Solution. We can use the command:

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normalcdf(starting value, ending value, mean, standard deviation)
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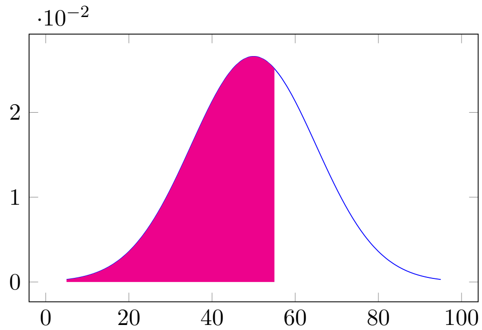
to obtain:

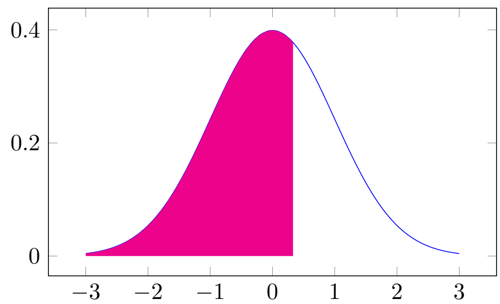
$$\texttt{normalcdf(-10E99,55,50,15)} = \boxed{0.6306}.$$

We can also transform the normal random variable into a standard normal random variable and then look up the values in a table:

$$P(X < 55) = P\left(Z < \frac{55-50}{15}\right) = P(Z < 1/3) \approx P(Z < 0.33) \approx \boxed{0.6293}.$$

Graphically this is:

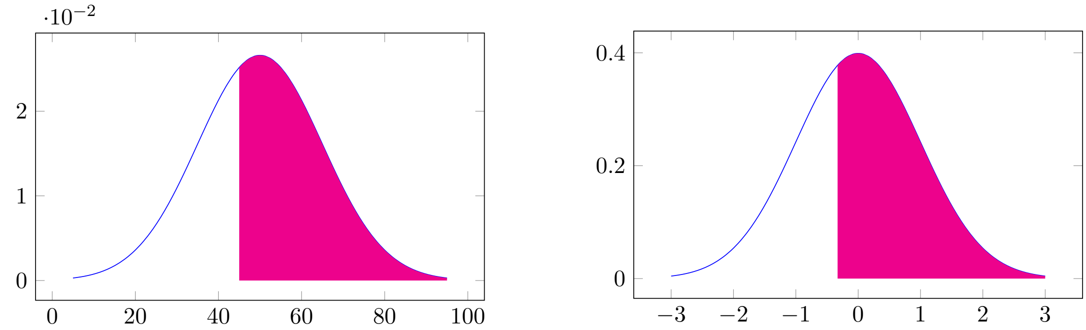




Since the normal distribution is symmetric, we have that:

$$\boxed{P(X > 45) = P(X < 55) \approx 0.6306 \approx 0.6293},$$

Graphically this is:



We can then compute:

$$\boxed{P(45 < X < 55) = P(X < 55) - (1 - P(45 < X)) \approx 0.2611 \approx 0.2586}.$$

Graphically this is: