NAME AND NETID:

Question 1. Let X be a normally distributed random variable measuring the average daily heat dissipation of a household. Given that X has mean $\mu = 4$ and $P(X \leq 5) = 0.88$, determine the probability that at least 3 units of energy are dissipated in a chosen day. (Hint: use that the normal distribution is symmetric about the mean) *[3]*

Solution. Recall that symmetry about the mean is the equality:

$$P(X \leq \mu - a) = P(X \geq \mu + a).$$

We then have:

$$P(X \geq 3) = P(X \geq 4 - 1) = P(X \leq 4 + 1) = P(X \leq 5) = \boxed{0.88}.$$

Question 2. Calculate the accumulated value after eight years on a principal of \$2700 given a nominal interest rate of 3.5% compounded every 6 months. *[3]*

Solution. The nominal rate of 0.035 is assumed to be annual, and thus there are two compounding periods every year. Over eight years, the units of time are 16 since compounding twice a year means that we have two units of time per year. Applying the formula we obtain that the accumulated value in dollars is:

$$\boxed{2700 \cdot \left(1 + \frac{0.035}{2}\right)^{16} \approx 3563.81}.$$

Question 3. The grades of students in a MATH141 class are normally distributed with mean $\mu = 74$ and variance $\sigma^2 = 38$. If X is the random variable denoting the grade value of a randomly chosen student, calculate $P(67 \leq X \leq 89)$ to four decimal places. *[4]*

Solution. We can use the command:

    normalcdf(starting value, ending value, mean, standard deviation)

to obtain:
$$\texttt{normalcdf(67,89,74,\textbackslash sqrt\{38\})} = \boxed{0.8644}.$$

We can also transform the normal random variable into a standard normal random variable and then look up the values in a table:

$$P(67 \leq X \leq 89) = P\left(\frac{89 - 74}{\sqrt{38}} \leq Z \leq \frac{89 - 74}{\sqrt{38}}\right) \approx P(-1.14 \leq Z \leq 2.43)$$

$$= P(Z \leq 2.43) - P(Z \leq -1.14) \approx 0.9925 - 0.1271 \approx \boxed{0.8654}.$$

Bonus Question. Suppose X is a normal random variable with $\mu = 120$ and $\sigma = 75$.

1. Find the values of $P(X < 65)$, $P(X > 170)$, and $P(50 < X < 145)$. **[3]**

2. Approximately draw this normal curve and make a sketch of the area under the curve corresponding to each of these probabilities. **[3]**

Solution.

1. We can use the command:

   `normalcdf(starting value, ending value, mean, standard deviation)`

   to obtain:

   $$\text{normalcdf}(-10\text{E}99, 65, 120, 75) = \boxed{0.2317}$$
   $$\text{normalcdf}(170, 10\text{E}99, 120, 75) = \boxed{0.2525}$$
   $$\text{normalcdf}(50, 145, 120, 75) = \boxed{0.4552}$$

   We can also transform the normal random variable into a standard normal random variable and then look up the values in a table:

   $$P(X < 65) = P\left(Z < \frac{65 - 120}{75}\right) \approx P(Z < -0.73) = \boxed{0.2327}$$
   $$P(170 < X) = P\left(\frac{170 - 120}{75} < Z\right) \approx P(0.67 < Z)\boxed{0.2514}$$
   $$P(50 < X < 145) = P\left(\frac{50 - 120}{75} < Z < \frac{145 - 120}{75}\right)$$
   $$\approx P(-0.93 < Z < 0.33) = \boxed{0.4531}$$

2. Graphically $P(X < 65)$ is magenta, $P(170 < X)$ cyan, and $P(50 < X < 145)$ is yellow: