

Question 1: There are one-sided inverses:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} -\frac{17}{18} & \frac{8}{18} \\ -\frac{2}{18} & \frac{2}{18} \\ \frac{13}{18} & -\frac{4}{18} \end{pmatrix}$$

These satisfy: $A \cdot B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ but $B \cdot A \neq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Question 2: $\log_b(x^n) = n \cdot \log_b(x)$ is equivalent to $(e^x)^n = e^{x \cdot n}$.

If we know that $(e^x)^n = e^{x \cdot n}$, then:

$$n \cdot \log_b(x) = \log_b(b^{n \cdot \log_b(x)}) = \log_b((b^{\log_b(x)})^n) = \log_b(x^n).$$

Question 3: $f(x) = \frac{3x+2}{5x-1}$ has inverse $g(x) = \frac{x+2}{5x-3}$.

$$f(g(x)) = \frac{3 \cdot \left(\frac{x+2}{5x-3}\right) + 2}{5 \cdot \left(\frac{x+2}{5x-3}\right) - 1} = \frac{\frac{3x+6}{5x-3} + \frac{10x-6}{5x-3}}{\frac{5x+10}{5x-3} - \frac{5x-3}{5x-3}} = \frac{13x}{13} = x$$

$$g(f(x)) = \frac{\frac{3x+2}{5x-1} + 2}{5 \cdot \left(\frac{3x+2}{5x-1}\right) - 3} = \frac{\frac{3x+2}{5x-1} + \frac{10x-2}{5x-1}}{\frac{15x+10}{5x-1} - \frac{15x-3}{5x-1}} = \frac{13x}{13} = x$$