

Example: Decompose into partial fractions $\frac{x^2+2}{2(x-1)(x-4)(x+2)}$.

$$\frac{x^2+2}{2(x-1)(x-4)(x+2)} = \frac{A}{x-1} + \frac{B}{x-4} + \frac{C}{x+2}$$

Multiply by $2(x-1)(x-4)(x+2)$ to clear denominators:

$$x^2+2 = 2A(x-4)(x+2) + 2B(x-1)(x+2) + 2C(x-1)(x-4)$$

To compute A , set $x=1$: $1^2+2 = 2 \cdot A \cdot (1-4)(1+2)$ so $A = \frac{-1}{6}$.

To compute B , set $x=4$: $4^2+2 = 2 \cdot B \cdot (4-1)(4+2)$ so $B = \frac{1}{2}$.

To compute C , set $x=-2$: $(-2)^2+2 = 2 \cdot C \cdot (-2-1)(-2-4)$ so $C = \frac{1}{6}$.

$$\text{Then: } \frac{x^2+2}{2x^3-6x^2-12x+16} = \frac{\frac{-1}{6}}{x-1} + \frac{\frac{1}{2}}{x-4} + \frac{\frac{1}{6}}{x+2} = \frac{-1}{6(x-1)} + \frac{1}{2(x-4)} + \frac{1}{6(x+2)}$$