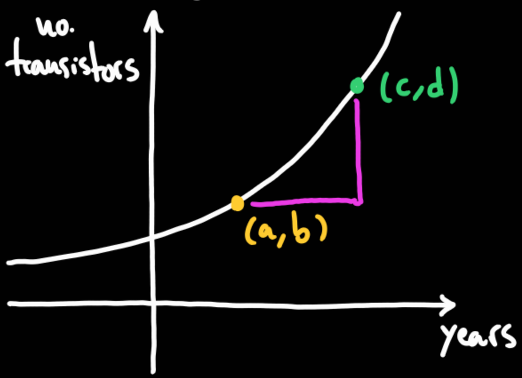
Problem 7.4.29.:

a) Estimate growth constant k:



$$y' = k \cdot y$$

$$\text{slope}: \frac{d-b}{c-a}.$$

Processor	Year	No. Transistors
...		
	1971	2250
	1972	2500

$a = (1971, 2250)$ $b = (1972, 2500)$

$$m = \frac{2500 - 2250}{1972 - 1971} = 250$$

$$P(t) = P_0 \cdot e^{k \cdot t}$$

$$\frac{d}{dt}(P(t)) = P_0 \cdot k \cdot e^{kt} = k \cdot P_0 \cdot e^{k \cdot t} = k \cdot P(t).$$

$$a: \; 2250 = P(1971) = P_0 \cdot e^{k \cdot 1971}$$

$$b: \; 2500 = P(1972) = P_0 \cdot e^{k \cdot 1972}$$

$$\frac{b}{a}: \; \frac{2500}{2250} = \frac{P_0 \cdot e^{k \cdot 1972}}{P_0 \cdot e^{k \cdot 1971}} = e^k \rightsquigarrow k = \ln\left(\frac{2500}{2250}\right) \approx 0.105$$

Processor	Year	No. Transistors

Processor	Year	No. Transistors
...		
	2000	42 000 000
	2008	1900 000 000

$$\frac{1900\,\cancel{000}\,\cancel{000}}{42\,\cancel{000}\,\cancel{000}} = \frac{\cancel{P_0}\cdot e^{k\cdot 2008}}{\cancel{P_0}\cdot e^{k\cdot 2000}} = e^{k\cdot 8} \rightsquigarrow k\cdot 8 = \ln\left(\frac{1900}{42}\right).$$

Problem 7.4.31: Find the doubling time of $f(t) = at+b$.

Fix t_0 real number. We want: $f(t_0+T) = 2\cdot f(t_0)$, T doubling time at t_0.

$$a\cdot(t_0+T)+b = f(t_0+T) = 2\cdot f(t_0) = 2\cdot(a\cdot t_0+b)$$

$$a\cdot t_0 + a\cdot T + b = 2\cdot a\cdot t_0 + 2\cdot b$$

$$a\cdot T = 2\cdot a\cdot t_0 - a\cdot t_0 + 2\cdot b - b = a\cdot t_0 + b \quad \text{so} \quad T = t_0 + \frac{b}{a}.$$

Problem 7.1.34: $\frac{d}{dt}(y) = k\cdot y\cdot \ln\left(\frac{y}{M}\right).$

Check that $y = M\cdot e^{a\cdot e^{k\cdot t}}$ satisfies the equation above.

$$\frac{d}{dt}(y) = \frac{d}{dt}\left(M\cdot e^{a\cdot e^{k\cdot t}}\right) = M\cdot\frac{d}{dt}\left(e^{a\cdot e^{k\cdot t}}\right) = M\cdot a\cdot k\cdot e^{k\cdot t}\cdot e^{a\cdot e^{k\cdot t}} =$$

$$= \underbrace{M\cdot a\cdot k\cdot e^{k\cdot t + a\cdot e^{k\cdot t}}}.$$

$$\boxed{\begin{array}{l} \frac{d}{dt}\left(e^{f(t)}\right) = f'(t)\cdot e^{f(t)}. \\ f(t) = a\cdot e^{k\cdot t},\quad f'(t) = a\cdot k\cdot e^{k\cdot t} \end{array}}$$

$$k\cdot y\cdot \ln\left(\frac{y}{M}\right) = k\cdot M\cdot e^{a\cdot e^{k\cdot t}}\cdot \ln\left(\frac{M\cdot e^{a\cdot e^{k\cdot t}}}{M}\right) =$$

$$= k\cdot M\cdot e^{a\cdot e^{k\cdot t}}\cdot \ln\left(e^{a\cdot e^{k\cdot t}}\right) = k\cdot M\cdot e^{a\cdot e^{k\cdot t}}\cdot a\cdot e^{k\cdot t} =$$

$$= \underbrace{k\cdot M\cdot a\cdot e^{a\cdot e^{k\cdot t}+k\cdot t}}.$$

$$\boxed{\begin{array}{l}\ln(e^x) = x \\ x = a\cdot e^{k\cdot t}\end{array}}$$

Problem 7.4.85: Assume $P(t) = M\cdot e^{a\cdot e^{k\cdot t}}$ with $M = 204$, $k = 0.15\ (\text{month})^{-1}$.

Find $P(t)$, noting that $P(0) = 200$. So:

$$200 = P(0) = 204\cdot e^{a\cdot e^{0.15\cdot 0}} = 204\cdot e^{a} \rightsquigarrow e^{a} = \frac{200}{204}$$

so $a = \ln\left(\frac{200}{204}\right) \approx -0.0198$. Then: $P(t) = 204\cdot e^{-0.0198\cdot e^{0.15\cdot t}}$.

Example 7.1.8.:

b) $$\int x\cdot e^{2x^2}\cdot dx = \int e^{2x^2}\cdot x\cdot dx = \frac{4}{4}\cdot\int e^{2x^2}\cdot x\cdot dx = \frac{1}{4}\int \underbrace{e^{2x^2}}_{e^u}\cdot \underbrace{4x\,dx}_{du} =$$

$u = 2x^2$

$\frac{du}{dx} = 4x \rightsquigarrow du = 4x\cdot dx$

$$= \frac{1}{4}\int e^{u}\cdot du = \frac{1}{4}e^{u} + C \underset{u=2x^2}{=} \frac{1}{4}e^{2x^2} + C.$$