Problem 7.7.46: Idea: $e^{\ln(f(x))} = f(x)$, so: $\lim_{x\to a} f(x) = \lim_{x\to a} e^{\ln(f(x))}$.

Also, we can put the limit inside the exponential:

$$\lim_{x\to a} f(x) = \lim_{x\to a} e^{\ln(f(x))} = e^{\lim_{x\to a} \ln(f(x))}.$$

this is allowed (because the exponential function is continuous).

$$\lim_{x\to\infty} x^{\frac{1}{x^2}} = e^{\lim_{x\to\infty} \ln\left(x^{\frac{1}{x^2}}\right)} = e^{\lim_{x\to\infty} \frac{1}{x^2}\cdot \ln(x)} = e^0 = 1.$$

$\ln\left(x^{\frac{1}{x^2}}\right) = \frac{1}{x^2}\cdot \ln(x)$ $\qquad$ $\lim_{x\to\infty} \frac{\ln(x)}{x^2} = 0$

apply LHR.

$e^x \gg x^n \gg \ln(x)$

Problem 7.8.53:

$$\int \frac{dx}{4x^2+1} = \int \frac{\frac{du}{2}}{u^2+1} = \frac{1}{2}\int \frac{du}{u^2+1} = \frac{1}{2}\cdot \arctan(u) + C_1 =$$

looks like $\frac{1}{x^2+1}$. We do not know how to integrate $\frac{1}{4x^2+1}$ but we do know how to integrate $\frac{1}{x^2+1}$:

$$\int \frac{dx}{x^2+1} = \arctan(x) + C_1 = \tan^{-1}(x) + C_1$$

p. 20 class notes

$u = 2x \qquad du = 2\,dx$

$$\frac{d}{dx}(\arctan(x)) = \frac{1}{x^2+1}.$$

$$= \frac{1}{2}\arctan(2x) + C_1$$

Problem 7.8.70: Fancy trig substitution.

$$\int \frac{\arctan(x)}{x^2+1}\,dx = \int u\,du = \frac{u^2}{2} + C_1 = \frac{\arctan^2(x)}{2} + C_1.$$

$u = \arctan(x)$

$du = \frac{1}{x^2+1}\,dx$

L logarithms
I inverse trig. functions
A algebraic functions
T trig functions
E exponentials

for choosing u.

Problem 7.8.98:

$$\int \frac{dx}{x \cdot (\ln(x))^5} = \int \frac{du}{u^5} = \frac{-1}{4u^4} + C_1 = \frac{-1}{4 \cdot (\ln(x))^4} + C_1.$$

$u = \ln(x)$ $\quad du = \frac{1}{x} \cdot dx$

$$\frac{du}{dx} = \frac{1}{x}$$