**Example 8.6.9.:** Does $\int_1^\infty \frac{dx}{\sqrt{x}+e^{3x}}$ converge?

Rule of thumb: in fractions, the bigger the denominator is, the more likely an integral converges.

$\frac{1}{\sqrt{x}+\underbrace{e^{3x}}_{\text{this gets big very fast}}}$ the integral of this should be convergent.

Comparison test: for $f(x) \geq g(x) \geq 0$, if $\underbrace{\int_a^\infty f(x)\,dx}_{\substack{\text{we need to} \\ \text{choose } f(x) \\ \text{bigger than } g(x) \\ \text{and with convergent integral.}}}$ converges, then $\underbrace{\int_a^\infty g(x)\,dx \text{ converges.}}_{\substack{\text{the comparison test says that} \\ \text{this converges, so:} \\ g(x) = \frac{1}{\sqrt{x}+e^{3x}}.}}$

$$g(x) = \frac{1}{\sqrt{x}+e^{3x}} \leq \frac{1}{\sqrt{x}}$$

making the denominator smaller yields a bigger fraction.

$$g(x) = \frac{1}{\sqrt{x}+e^{3x}} \leq \frac{1}{e^{3x}}$$

Note $x \geq 1$ by the limits of integration.

Note $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ is a p-integral that does not converge. So $f(x) = \frac{1}{\sqrt{x}}$ is not a good choice.

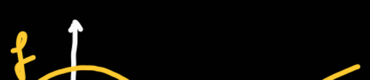
However $\int_1^\infty \frac{1}{e^{3x}}\,dx = \frac{1}{3e^3}$ is convergent. So choosing $f(x) = \frac{1}{e^{3x}}$ the comparison test

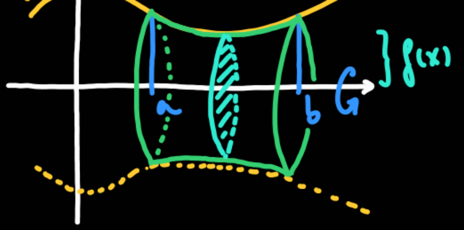
gives convergence for $\int_1^\infty \frac{dx}{\sqrt{x}+e^{3x}}$.

**Problem 8.6.82.:** Compute the volume of the solid obtained by rotating $y = e^{-\frac{|x|}{2}}$ about the x-axis

for $-\infty < x < \infty$.

## Section 6.3.: Volumes of revolution.



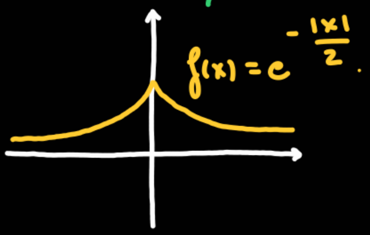


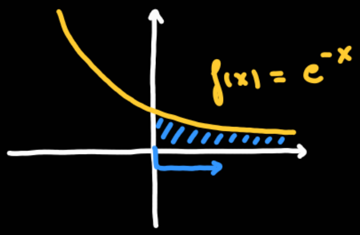

Disk gives area $\pi(f(x))^2$

We then integrate this from a to b.

$$V=\int_a^b \pi (f(x))^2 dx = \pi\int_a^b (f(x))^2 dx.$$

$$V=\pi\int_{-\infty}^{\infty}(f(x))^2 dx = 2\pi\int_0^{\infty}\left(e^{-\frac{x}{2}}\right)^2 = 2\pi\int_0^{\infty} e^{-x} dx = \cdots = 2\pi.$$

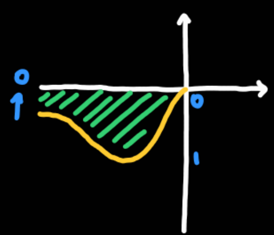


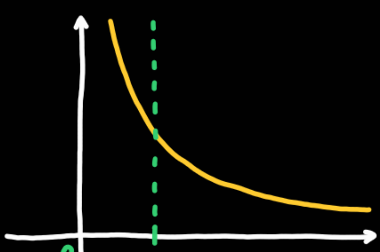

**Problem 8.6.10.:** Compute $\int_0^5 \frac{dx}{x^{\frac{19}{20}}} = 20\cdot 5^{\frac{1}{20}}$


**Problem 8.6.11.:** Compute $\int_0^4 \frac{dx}{\sqrt{4-x}} = 4.$

**Problem 8.6.26.:** Compute $\int_{-\infty}^0 x\cdot e^{-x^2} dx = \frac{-1}{2}.$




**Problem 8.6.73.:** Check divergence or convergence of $\int_0^{\infty}\frac{dx}{(x+x^2)^{\frac{1}{3}}}$.



for $x\to\infty$ : $\frac{1}{(x+x^2)^{\frac{1}{3}}} \rightsquigarrow \frac{1}{(x^2)^{\frac{1}{3}}} = \frac{1}{x^{2/3}}$ looks like divergent.

for $x\to 0$ : $\frac{1}{(x+x^2)^{\frac{1}{3}}} \rightsquigarrow \frac{1}{x^{\frac{1}{3}}}$ looks like convergent.

(p-integral with $p<1$ from 0 to 1)

Now: $\displaystyle\int_0^\infty \frac{dx}{(x+x^2)^{1/3}} = \int_0^1 \frac{dx}{(x+x^2)^{1/3}} + \int_1^\infty \frac{dx}{(x+x^2)^{1/3}}$

$\displaystyle\int_0^1 \frac{dx}{(x+x^2)^{1/3}} \leq \int_0^1 \frac{dx}{x^{1/3}}$ which converges.

$\displaystyle\frac{1}{(x+x^2)^{1/3}} \leq \frac{1}{x^{1/3}}$

Careful: ⚠ $\displaystyle\int_1^\infty \frac{dx}{(x+x^2)^{1/3}} \leq \int_1^\infty \frac{dx}{x^{2/3}}$ does <u>not</u> give us any useful information.

However: $\displaystyle\frac{1}{(x+x^2)^{1/3}} \geq \frac{1}{2^{1/3}x^{2/3}}$. Now: $\displaystyle\int_1^\infty \frac{dx}{(x+x^2)^{1/3}} \geq \int_1^\infty \frac{dx}{2^{1/3}x^{2/3}}$ which diverges.

for $x \geq 1$ we have $x \leq x^2$ so $x+x^2 \leq x^2+x^2 = 2x^2$ so $\displaystyle\frac{1}{2x^2} \leq \frac{1}{x+x^2}$

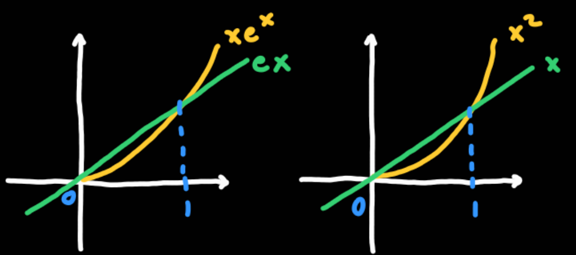
<u>Problem 8.6.74.:</u> Check whether $\displaystyle\int_0^\infty \frac{dx}{xe^x+x^2}$ converges or diverges.

A similar thing as in the previous problem happens:

$$\int_0^\infty \frac{dx}{xe^x+x^2} = \underbrace{\int_0^1 \frac{dx}{xe^x+x^2}}_{\frac{1}{x^2} \text{ and } \frac{1}{xe^x} \text{ do not give information.}} + \underbrace{\int_1^\infty \frac{dx}{xe^x+x^2}}_{\text{converges without issues}}$$

$\displaystyle\int_1^\infty \frac{dx}{xe^x+x^2} \leq \int_1^\infty \frac{dx}{x^2}$ which converges.

However for $0 \leq x \leq 1$ we have $xe^x \leq ex$ and $x^2 \leq x$. Then $xe^x + x^2 \leq ex + x = (e+1)x$.




So: $\displaystyle\frac{1}{(e+1)x} \leq \frac{1}{xe^x+x^2}$.

Now: $\displaystyle\int_0^1 \frac{dx}{xe^x+x^2} \geq \int_0^1 \frac{dx}{(e+1)x}$ which diverges.