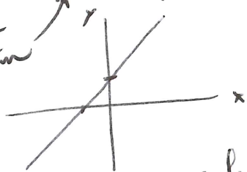
Linear algebra: $\underbrace{\text{linear equations}}_{\text{today}}$ and $\underbrace{\text{linear transformations}}_{\text{in a couple of weeks}}$.

coefficients ↓ ↙ variables

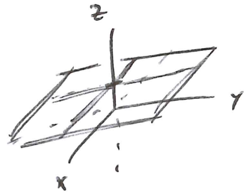
$y = x + 1 \rightsquigarrow y - x = 1$

constant term



with more variables: $x + y + z = 1$

$z = 1$



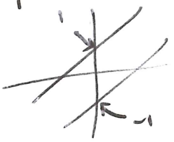
Drawing them is finding solutions!

What if we have more than one?

We have system of eq.:

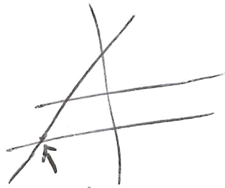
$y - x = 1$

$y - x = -1$



$y - x = 1$

$y = -1$



$(-2, -1)$

For more complicated ones;

$$\begin{cases} x + 2y + 3z = 6 \\ 2x - 3y + 2z = 14 \\ 3x + y - z = -2 \end{cases} \xrightarrow{R_2 - 2R_1} \begin{cases} x + 2y + 3z = 6 \\ -7y - 4z = 2 \\ 3x + y - z = -2 \end{cases} \xrightarrow{R_3 - 3R_1}$$

$$\begin{cases} x + 2y + 3z = 6 \\ -7y - 4z = 2 \\ -5y - 10z = -20 \end{cases} \xrightarrow{R_2 \leftrightarrow R_3} \begin{cases} x + 2y + 3z = 6 \\ -5y - 10z = -20 \\ -7y - 4z = 2 \end{cases} \xrightarrow{\frac{R_2}{-5}}$$

$$\begin{cases} x + 2y + 3z = 6 \\ y + 2z = 4 \\ -7y - 4z = 2 \end{cases} \xrightarrow{R_3 + 7R_2} \begin{cases} x + 2y + 3z = 6 & \rightsquigarrow x = 1 \\ y + 2z = 4 & \rightsquigarrow y = -2 \\ 10z = 30 & \rightsquigarrow z = 3 \end{cases}$$

It turns out, by only doing these three things, we can always solve these systems!

Let's rephrase what we have done into matrix notation.

A matrix is just a way of organizing numbers:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \begin{array}{c} 2 \times 3 \\ \uparrow \;\; \uparrow \\ \text{row} \;\; \text{column} \end{array}$$

We can write a system of equations into an <u>augmented matrix</u>

$$\begin{cases} x + 2y + 3z = 6 \\ 2x - 3y + 2z = 14 \\ 3x + y - z = -2 \end{cases} \rightsquigarrow \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & -3 & 2 & 14 \\ 3 & 1 & -1 & -2 \end{array}\right] \\ \begin{array}{cccc} x & y & z & = \end{array} \end{array}$$

linear terms.

We do <u>row</u> operations because these correspond to operations on the <u>equations</u>. Do <u>not</u> do column operations!

What have we done?

1. Divide a row by a non-zero scalar.
2. Subtract a multiple of a row from another row.
3. Swap two rows.

This got us to:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 10 & 30 \end{array}\right]$$

where we could read the solution. We can do even better!

$$\downarrow R_3/10$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_2 - 2R_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1 - 3R_3} \left[\begin{array}{ccc|c} 1 & 2 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{R_1 - 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

solution!

this most simplified form is called <u>row reduced echelon form</u>.

<u>RREF</u>:

(pivot) (leading 1)

(i) If a row has non-zero entries, then the first non-zero entry is a 1.

(ii) If a column contains a leading one, then all the other entries in the column are 0.

(iii) If a row contains a leading one, then each row above it contains a leading one further to the left.

<u>Gauss-Jordan elimination</u>: bring a matrix A to rref(A) it's row reduced echelon form.