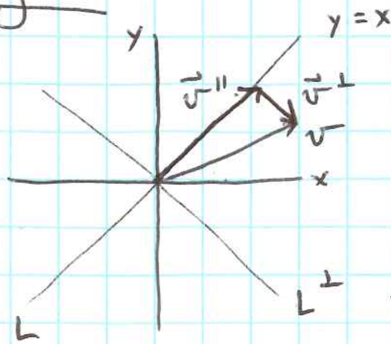
Projection:



$$\vec{u} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\vec{v}^{\parallel} = \text{proj}_L \vec{v} = (\vec{v} \cdot \vec{u})\vec{u}$$

$\begin{bmatrix} a \\ b \end{bmatrix}$ perpendicular to $\vec{u}$, solve $\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = 0$ and normalize.

$$\vec{v}^{\perp} = \text{proj}_{L^{\perp}} \vec{v} = (\vec{v} \cdot \vec{w})\vec{w} = \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix}$$

Indeed: $\vec{v} = \vec{v}^{\parallel} + \vec{v}^{\perp}$.

Matrix of projection:

$$\text{proj}_L \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \qquad \text{proj}_L \vec{v} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}\vec{v}.$$

$$\text{proj}_L \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

Reflection:

$$\text{refl}_L \vec{v} = \vec{v} - 2\cdot\vec{v}^{\perp} = \vec{v} - 2\cdot(\vec{v} - \vec{v}^{\parallel}) =$$

$$\text{refl}_L \vec{v} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$= 2\vec{v}^{\parallel} - \vec{v} = 2\cdot \text{proj}_L \vec{v} - \vec{v} =$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\vec{v} - \vec{v} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\vec{v} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\vec{v} =$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\vec{v}.$$

Composition of linear transformations is multiplication of matrices:

$$\mathbb{R}^2 \xrightarrow{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} \mathbb{R}^2 \xrightarrow{\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}} \mathbb{R}^2$$

$$\begin{array}{ccc} x_1 & \quad y_1 \quad & z_1 \\ x_2 & \quad y_2 \quad & z_2 \end{array}$$

$$① \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} ②$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} ③$$

$$\begin{cases} y_1 = x_1 + 2x_2 \\ y_2 = 3x_1 + 4x_2 \end{cases} \qquad \begin{cases} z_1 = 5y_1 + 6y_2 \\ z_2 = 7y_1 + 8y_2 \end{cases}$$

$$③'' \begin{cases} z_1 = 5(x_1 + 2x_2) + 6(3x_1 + 4x_2) = 23x_1 + 34x_2 \\ z_2 = 7(x_1 + 4x_2) + 8\cdot(3x_1 + 4x_2) = 31x_1 + 46x_2. \end{cases}$$

Composition of linear transformations is a linear transformation.