Math 33A
Linear Algebra and Applications

Discussion 3

Problem 1.

Show that if a square matrix A has two equal columns, then A is not invertible.

Problem 2($\star$).

Which of the following linear transformations T from $\mathbb{R}^3$ to $\mathbb{R}^3$ are invertible? Find the inverse if it exists.

(a) Reflection about a plane.

(b) Orthogonal projection onto a plane.

(c) Scaling by a real factor (namely, fix a real number r and consider $T(\vec{v}) = r\vec{v}$, for all vectors $\vec{v}$).

(d) Rotation about an axis.

Problem 3.

A square matrix is called a permutation matrix if it contains a 1 exactly once in each row and in each column, with all other entries being 0. Give an example of two different 3×3 permutation matrices.

Problem 4.

Are permutation matrices invertible? If so, is the inverse a permutation matrix as well?

Problem 5.

Consider two invertible $n \times n$ matrices A and B. Is the linear transformation $\vec{y} = A(B(\vec{x}))$ invertible? If so, what is the inverse?

Problem 6.

Are the columns of an invertible matrix linearly independent?

Problem 7.

Consider linearly independent vectors $\vec{v_1}, \ldots, \vec{v_m}$ in $\mathbb{R}^n$, and let A be an invertible $m \times m$ matrix. Are the columns of the following matrix linearly independent?

$$\begin{bmatrix} | & & | \\ \vec{v_1} & \cdots & \vec{v_m} \\ | & & | \end{bmatrix} A$$