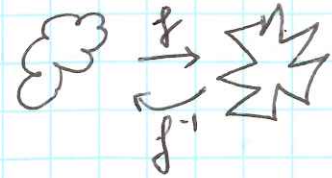
Recall: invertible functions:



A matrix is invertible when multiplication (on the left) by this matrix is an invertible linear transformation / function.

Question: invertible. How many solutions of $\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}5\\6\end{bmatrix}$?

$\begin{bmatrix}1&2\\3&4\end{bmatrix}$ $\quad \begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1&2\\3&4\end{bmatrix}^{-1}\begin{bmatrix}5\\6\end{bmatrix}$ one.

How to find inverses: Gauss-Jordan:

$$\left[\begin{array}{ccc|ccc}1&2&0&1&0&0\\2&1&1&0&1&0\\-1&0&-1&0&0&1\end{array}\right]\xrightarrow{R_2-2R_1}\left[\begin{array}{ccc|ccc}1&2&0&1&0&0\\0&-3&1&-2&1&0\\-1&0&-1&0&0&1\end{array}\right]$$

$$\xrightarrow{R_3+R_1}\left[\begin{array}{ccc|ccc}1&2&0&1&0&0\\0&-3&1&-2&1&0\\0&2&-1&1&0&1\end{array}\right]\xrightarrow{R_1-R_3}\left[\begin{array}{ccc|ccc}1&0&+1&0&0&-1\\0&-3&1&-2&1&0\\0&2&-1&1&0&1\end{array}\right]$$

$$\xrightarrow{R_2/-3}\left[\begin{array}{ccc|ccc}1&0&+1&0&0&-1\\0&1&-1/3&+2/3&-1/3&0\\0&2&-1&1&0&1\end{array}\right]\xrightarrow{R_3-2R_2}\left[\begin{array}{ccc|ccc}1&0&+1&0&0&-1\\0&1&-1/3&+2/3&-1/3&0\\0&0&-1/3&-1/3&+2/3&1\end{array}\right]$$

$$\xrightarrow{R_2-R_3}\left[\begin{array}{ccc|ccc}1&0&+1&0&0&-1\\0&1&0&1&-1&-1\\0&0&-1/3&-1/3&+2/3&1\end{array}\right]\xrightarrow{R_3\cdot(-3)}\left[\begin{array}{ccc|ccc}1&0&1&0&0&-1\\0&1&0&1&-1&-1\\0&0&1&+1&-2&-3\end{array}\right]$$

$$\xrightarrow{R_1-R_3}\left[\begin{array}{ccc|ccc}1&0&0&-1&2&2\\0&1&0&1&-1&-1\\0&0&1&1&-2&-3\end{array}\right].$$

Check!

$$\begin{bmatrix}1&2&0\\2&1&1\\-1&0&-1\end{bmatrix}\begin{bmatrix}-1&2&2\\1&-1&-1\\1&-2&-3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

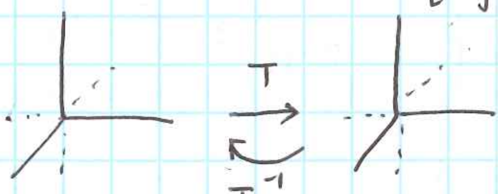
$$\begin{bmatrix}-1&2&2\\1&-1&-1\\1&-2&-3\end{bmatrix}\begin{bmatrix}1&2&0\\2&1&1\\-1&0&-1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Why does this work? Look at each column after the bar: $\left[\begin{array}{ccc|c}1&2&0&1\\2&1&1&0\\-1&0&-1&0\end{array}\right]$, $\left[\begin{array}{ccc|c}1&2&0&0\\2&1&1&1\\-1&0&-1&0\end{array}\right]$, + one.

We are <u>solving</u> the equation: $A\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, + two.

We are <u>finding</u> the vectors $\begin{bmatrix}x\\y\\z\end{bmatrix}$ that go to $\begin{bmatrix}1\\0\\0\end{bmatrix}$...

We are <u>computing</u> $T^{-1}\begin{bmatrix}1\\0\\0\end{bmatrix}$, $T^{-1}\begin{bmatrix}0\\1\\0\end{bmatrix}$, $T^{-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$.



The matrix associated to T^{-1} is:

$$\begin{bmatrix}T^{-1}\begin{bmatrix}1\\0\\0\end{bmatrix} & T^{-1}\begin{bmatrix}0\\1\\0\end{bmatrix} & T^{-1}\begin{bmatrix}0\\0\\1\end{bmatrix}\end{bmatrix}$$

, which is what we did!

<u>Question</u>: If A, B invertible, is AB invertible?

<u>Yes</u>: $(AB)^{-1} = B^{-1}A^{-1}$.

<u>Why is T^{-1} linear?</u>

$$T^{-1}(\vec{v}+\vec{w}) = T^{-1}(T(\vec{x})+T(\vec{y})) = \vec{x}+\vec{y} = T^{-1}(\vec{v})+T^{-1}(\vec{w})$$