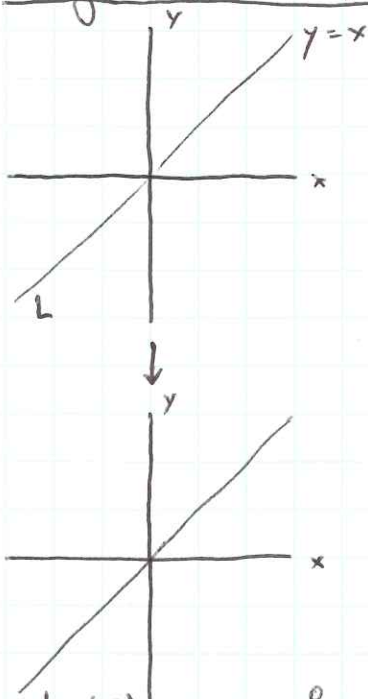
Image and kernel of a linear transformation:



projection onto L:

multiplication by $\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$.

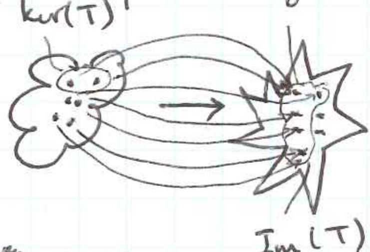
$T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$\mathrm{Im}(T) = \{\vec{y} \text{ in } \mathbb{R}^2 \text{ such that there is } \vec{x} \text{ in } \mathbb{R}^2 \text{ with } T(\vec{x}) = \vec{y}\} =$

$= \{T(\vec{x}) \text{ for } \vec{x} \text{ in } \mathbb{R}^2\}.$

$\ker(T) = \{\vec{x} \text{ in } \mathbb{R}^2 \text{ such that } T(x) = 0\} =$

$= \{\text{solutions of } A\vec{x} = 0\}.$



Eyeball ~~Compute~~ image: L.

Eyeball ~~Compute~~ kernel: $L^{\perp}$.

Compute image:

$$\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \underbrace{\frac{1}{2}x_1\begin{bmatrix}1\\ 1\end{bmatrix} + \frac{1}{2}x_2\begin{bmatrix}1\\ 1\end{bmatrix}}_{\text{span of columns!}} = (x_1 + x_2)\cdot\frac{1}{2}\begin{bmatrix}1\\ 1\end{bmatrix}$$

Compute kernel:

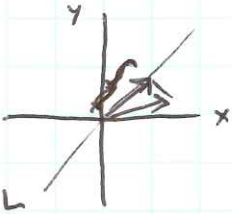
$$\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = 0 \iff x_1 + x_2 = 0 \iff x_1 = -x_2$$

$$\mathrm{Im}(T) = \left\{k\cdot\frac{1}{2}\begin{bmatrix}1\\ 1\end{bmatrix} \text{ for } k \text{ real number}\right\}$$

$$\ker(T) = \left\{k\cdot\begin{bmatrix}1\\ -1\end{bmatrix} \text{ for } k \text{ real number}\right\}$$

In general; Im(T) is all possible linear combinations of its columns. This is called "span".

The <u>span</u> of vectors $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}$ is <u>all</u> possible linear combinations of these vectors.



$$\xrightarrow[\text{of proj onto } L]{\text{length}} \quad \mathbb{R}$$

$$\vec{v} \overset{S}{\longmapsto} \vec{u} \cdot \vec{v} \quad \text{it is linear!}$$

Discussion session!

$$\frac{1}{2}\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \underbrace{\frac{1}{2}(x_1 + x_2)}_{\text{span of columns!}}$$

$\mathrm{Im}(S) = \{k \cdot [1] \text{ with } k \text{ real}\}$

$\ker(S) = \{k \cdot \frac{1}{2}\begin{bmatrix}1\\1\end{bmatrix}, k \text{ real number}\}$

In general, finding $\ker(T)$ is solving a system of eqs!

<u>Example 11 p. 117.</u>

Image and kernel are <u>vector spaces</u>:

(i) $\vec{0} \in \mathrm{Im}(T)$, $\vec{0} \in \ker(T)$.

(ii) Closed under addition.

(iii) Closed under multiplication by scalars.

A $n \times n$.
A invertible
if and only if
$\ker(A) = \{\vec{0}\}$.