Applications (and complex eigenvalues)

Question: Let A be a matrix with 0 not an eigenvalue.
Is A invertible? Yes! $\det(A - 0\cdot I_n) \neq 0$
$\Leftrightarrow$ A invertible.

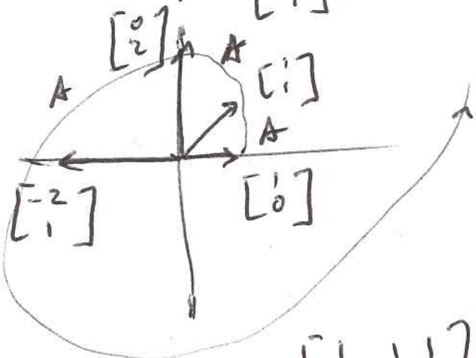
Question: Suppose A matrix with $f_A(x) = x^2 + x + 1$.
Is A invertible? Yes! $f_A(0) = 1 \neq 0$, not a root.

Exercise: Find the diagonalization of $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = A$.

Eigenvalues: $1 \pm i$.

$E_{1+i} = \text{span}\begin{bmatrix} i \\ 1 \end{bmatrix}$ $\qquad S^{-1}AS = \begin{bmatrix} 1+i & 0 \\ 0 & 1-i \end{bmatrix}$.

$E_{1-i} = \text{span}\begin{bmatrix} -i \\ 1 \end{bmatrix}$



Exercise: Diagonalize $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$. Find all eigenspaces / eigenvalues / eigenvectors.

$S = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ $\quad D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$S^{-1}AS = D$.

$S^{-1} = \begin{bmatrix} -1 & 1 & 2 \\ 2 & -2 & 1 \\ 3 & 3 & 0 \end{bmatrix} \cdot \frac{1}{6}$

Exercise: Compute A^{321}.

$A^{321} = S D^{321} S^{-1} = \begin{bmatrix} \text{—} \end{bmatrix}$.

Exercise: Compute: $\lim_{n\to\infty} \begin{bmatrix} 5/12 & 1/12 \\ 1/12 & 5/12 \end{bmatrix}^n$.

$$B = \begin{bmatrix} 5/12 & 1/12 \\ 1/12 & 5/12 \end{bmatrix}$$

$$S = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \qquad S^{-1} = \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

$$\lim_{n\to\infty} B^n = 0.$$

Question: Let A and B be similar.

Does $f_A(x) = f_B(x)$?

Does $\det(A) = \det(B)$?

Does $tr(A) = tr(B)$?

trace and determinant are coefficients of the characteristic polynomial.

$$f_A(0) = \det(A).$$
$$f_B(0) = \det(B).$$

$$f_A(x) = (-x)^n + tr(A)\cdot(-x)^{n-1} + \cdots$$
$$f_B(x) = (-x)^n + tr(B)\cdot(-x)^{n-1} + \cdots$$