

Math 31B
Integration and Infinite Series

Midterm 1

Instructions: You have 50 minutes to complete this exam. There are 6 questions, worth a total of 100 points. This test is closed book and closed notes. No calculator is allowed. Please write your solutions in the space provided, box your final answer, show all your work legibly, and clearly reference any theorems or results that you use. Do not forget to write your name, section (if you do not know your section, please write the name of your TA), and UID in the space below. Once the 50 minutes have elapsed, you are not allowed to continue writing and you are not allowed to communicate with anybody except the administrators of the exam. Please follow their requests at all times. **Failure to comply with any of these instructions may have repercussions in your final grade.**

Name: _____

ID number: _____

Section: _____

Question	Points	Score
1	15	
2	17	
3	17	
4	17	
5	17	
6	17	
Total:	100	

Problem 1. *15pts.*

Determine whether the following statements are true or false. If the statement is true, write **T** in the box provided under the statement. If the statement is false, write **F** in the box provided under the statement. Do not write “true” or “false”.

- (a) **F** The half-life of the exponential function $P(t) = e^t$ is $T = \ln(2)$.
- (b) **F** Let $f(x)$ be any function. Then $f'(x)$ exists.
- (c) **F** The derivative of $f(x) = b^x$ is $f'(x) = \ln(b)b^x$ for all real numbers b .
- (d) **T** The limit of $\frac{1}{x}$ when x approaches 0 does not exist.
- (e) **T** The hyperbolic function $\cosh(x)$ is always positive.

Problem 2. 17pts.

Find the inverse of $f(x) = \frac{1}{2x-5}$ at $x = 2$. Simplify your answer.

Solution: We have:

$$f^{-1}(x) = \frac{5x + 1}{2x}$$

for $x \neq 0$. The domain of $f(x)$ is all real numbers different from $\frac{5}{2}$. The range of $f(x)$ is all real numbers different from 0. Also:

$$f^{-1}(1) = \frac{11}{4}.$$

Problem 3. 17pts.

Find the derivative of $f(x) = x^{9x}$ at $x = \frac{1}{9}$. Simplify your answer.

Solution: We have:

$$f'(x) = 9x^{9x}(\ln(x) + 1)$$

and:

$$f'\left(\frac{1}{9}\right) = 1 - \ln(9).$$

Problem 4. 17pts.

Find the limit of $f(x) = \sec(x) - \tan(x)$ at $x = \frac{\pi}{2}$.

Solution: We have:

$$\lim_{x \rightarrow \pi/2} \sec(x) - \tan(x) = \lim_{x \rightarrow \pi/2} \frac{1 - \sin(x)}{\cos(x)} = 0.$$

Problem 5. 17pts.

(a) Find the integral of $f(x) = \frac{1}{\sqrt{1-16x^2}}$ between 0 and $\frac{1}{4}$. Simplify your answer.

(b) Find the integral of $f(x) = e^{-x} \sinh(x)$ between 0 and $\frac{1}{2}$. Simplify your answer.

Solution:

(a) We have:

$$\int_0^{1/4} \frac{dx}{\sqrt{1-16x^2}} = \frac{\arcsin(4x)}{4} \Big|_0^{1/4} = \frac{\arcsin(1)}{4} - \frac{\arcsin(0)}{4} = \frac{\frac{\pi}{2}}{4} - 0 = \frac{\pi}{8}.$$

(b) We have:

$$\int_0^{1/2} e^{-x} \sinh(x) dx = \int_0^{1/2} \frac{1 - e^{-2x}}{2} dx = \frac{x}{2} + \frac{e^{-2x}}{4} \Big|_0^{1/2} = \frac{1}{4e}.$$

Problem 6. 17pts.

Find the integral of $f(x) = x \ln(x)$ between 1 and 2. Simplify your answer.

Solution: We have:

$$\int_1^2 x \ln(x) dx = x^2 \left(\frac{\ln(x)}{2} - \frac{1}{4} \right) \Big|_1^2 = \ln(4) - \frac{3}{4}.$$