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Homework	25%		
Discussions	15%		
Midterm 1	15%	25%	
Midterm 2	15%		
Final	30%	35%	60%

Rogawski: 2nd.

Section 7.4.: Exponential growth and decay

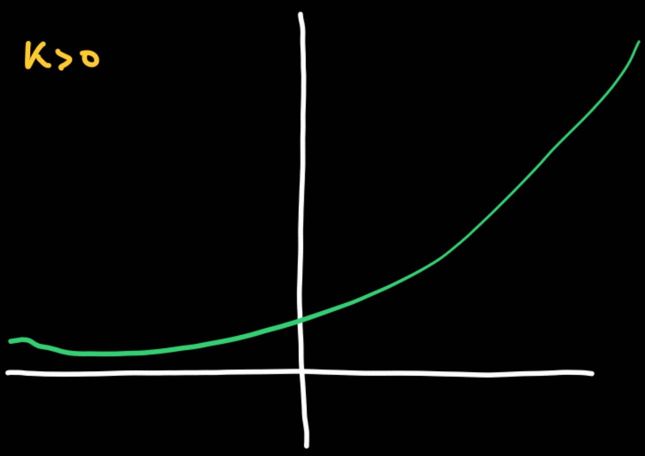
$$P(t) = \underbrace{P_0}_{\text{constant}} \cdot e^{\overbrace{k}^{\text{constant}} \cdot t}$$

k any real number $k>0$

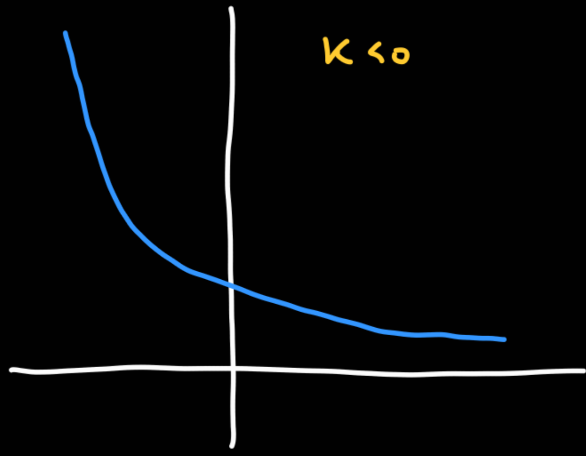
$k<0$

P_0 positive

$$P(0) = P_0 \cdot e^{k \cdot 0} = P_0 \cdot e^0 = P_0$$



exponential growth



exponential decay.

Example: Bacteria growing: $K = 0.41 \frac{1}{\text{hours}}$

We have 1000 bacteria at $t = 0$.

a) Find the population:

$$P(t) = P_0 \cdot e^{Kt} = P_0 \cdot e^{0.41 \cdot t}$$

$$P_0 = \frac{P(t)}{e^{0.41 \cdot t}} = \frac{P(0)}{1} = 1000$$

$$P(t) = 1000 \cdot e^{0.41 \cdot t}$$

b) Find the population after 5 hours.

$$P(5) = 1000 \cdot e^{0.41 \cdot 5} \approx 7765$$

c) What is the time required to obtain 10000 bacteria.

$$10\cancel{000} = P(t) = 1\cancel{000}\, e^{0.41 t}$$

$$e^{0.41 \cdot t} = 10 \xrightarrow{\text{apply ln}} 0.41 \cdot t = \ln(10) \qquad t = \frac{\ln(10)}{0.41} \approx 5.62 \text{ hours}$$

Rate of growth of a function.

$$\frac{d\,P(t)}{dt} = c \cdot P(t)$$

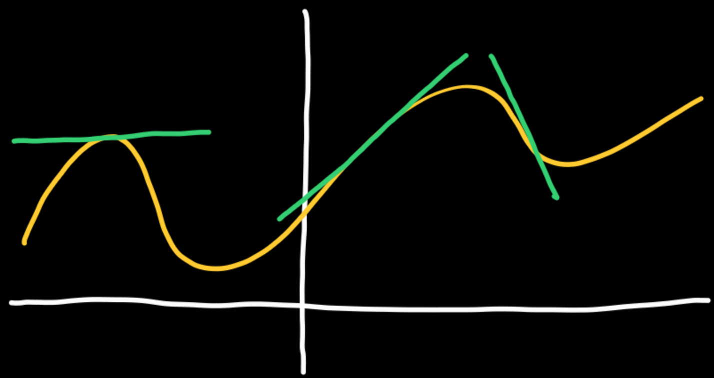
If $f(t)$ is a function

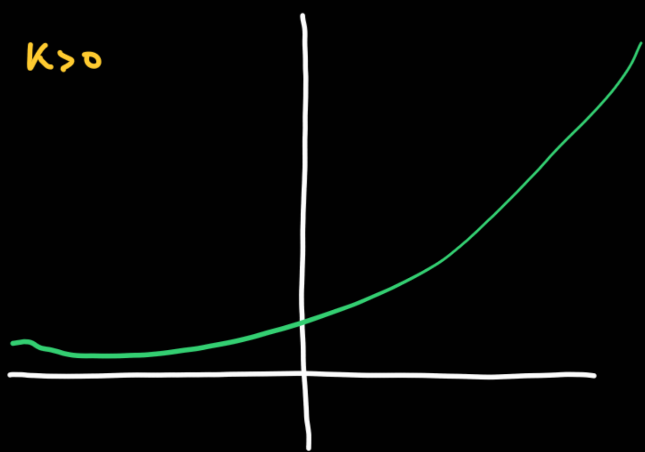
with $\frac{d\,f(t)}{dt} = c \cdot f(t)$

then $f(t)$ is an exponential.

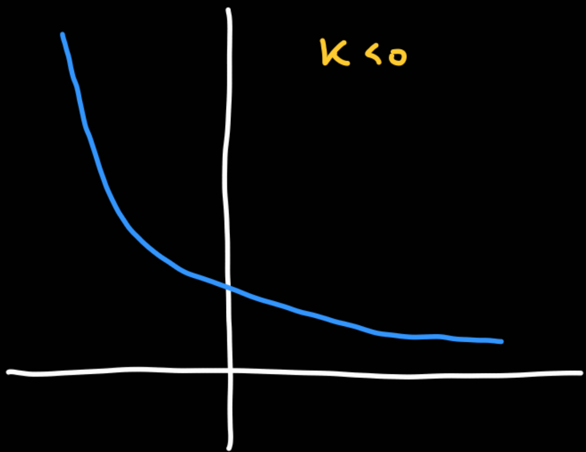
Derivatives

"instantaneous rate of growth"





exponential growth



exponential decay.

Doubling time:

$K>0 \quad T$

$$\boxed{T = \frac{\ln(2)}{k}}$$

$$P(t+T) = 2 \cdot P(t)$$

↑ find T

$$\cancel{P_0} \cdot e^{k\cdot(t+T)} = 2 \cdot \cancel{P_0} \cdot e^{kt}$$

$$2e^{kt} = e^{k(t+T)} \longleftarrow \frac{1}{e^{k\cdot(t+T)}} = \frac{1}{2e^{k\cdot t}}$$

Half-life:

$k<0 \quad T$

$$P(t+T) = \frac{1}{2} \cdot P(t)$$

↑ find T

$$\cancel{P_0} \cdot e^{-k\cdot(t+T)} = \frac{1}{2} \cancel{P_0} \cdot e^{-kt}$$

$P(t) = P_0 \cdot e^{kt}$

$k<0$

$P(t) = P_0 \cdot e^{-kt}$

$k>0$