Doubling time: $k>0$

$$P(t+T) = 2\cdot P(t) \rightsquigarrow T = \frac{\ln(2)}{k}$$

$$\cancel{P_0}\cdot e^{k\cdot(t+T)} = 2\cdot \cancel{P_0}\cdot e^{kt}$$

$$e^{kt+kT} = 2e^{kt}$$

$$\cancel{e^{kt}}\cdot e^{kT} = 2\cancel{e^{kt}}$$

$$e^{kT} = 2 \rightsquigarrow kT = \ln(2) \rightsquigarrow T = \frac{\ln(2)}{k}$$

Half-life: $k<0$

$$P(t+T) = \frac{1}{2}P(t) \rightsquigarrow T = \frac{\ln(2)}{k}$$

$$P(t) = P_0\cdot e^{kt} \quad k<0$$

$$P(t) = P_0\cdot e^{-kt} \quad k>0$$

$$\cancel{P_0}\cdot e^{-k(t+T)} = \frac{1}{2}\cancel{P_0}e^{-kt}$$

$$e^{-k(t+T)} = \frac{1}{2}e^{-kt}$$

$$\frac{1}{e^{k(t+T)}} = \frac{1}{2e^{kt}}$$

$$2e^{kt} = e^{k(t+T)}$$