MATH 33A.

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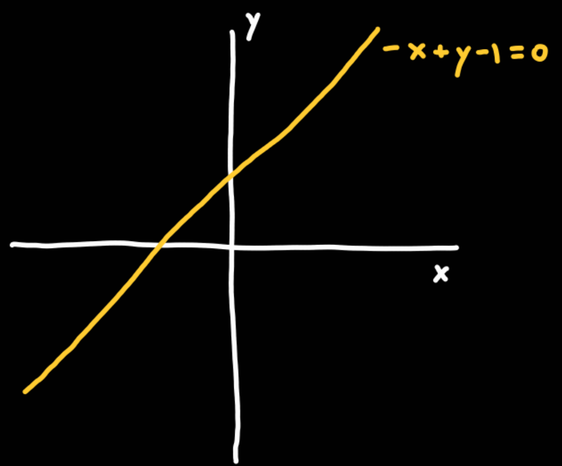
Syllabus: Grade 1 Grade 2 Grade 3

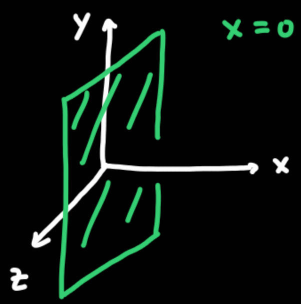
1. Introduction: (Chapter 1, Chapter 2)

Linear algebra studies <u>linear equations</u> and <u>linear transformations</u>.

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n + b = 0$$

coefficients variables constant term

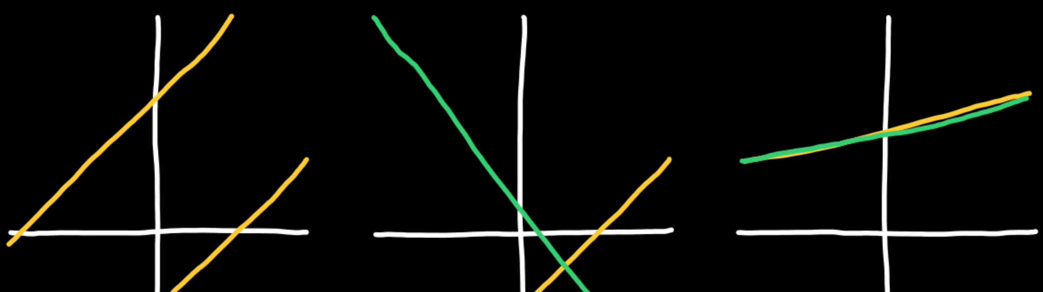




Systems of linear equations can have no solution, one solution, or infinitely many solutions.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + b_1 = 0$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + b_n = 0$$

row
a_{ij}
column



Matrices, matrix:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$ is an $\overset{\text{row}}{n} \times \underset{\text{column}}{m}$ matrix with entries a_{ij}.

A <u>matrix</u> is a rectangular array of numbers. If a matrix has n rows and m columns, we say that it has <u>size</u> $n \times m$. Two matrices A, B are equal when their entries a_{ij}, b_{ij} are equal.

Some families of matrices have names:

(i) Square matrices $(n \times n)$

(ii) Diagonal matrix (everything outside the diagonal is zero, i.e. $\overset{a_{ii}}{a_{ij}} = 0$ for $i \neq j$).

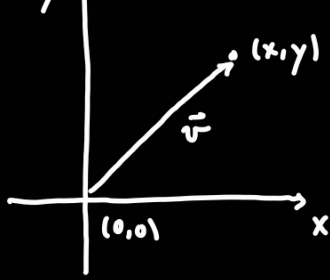
(iii) Upper triangular matrices $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

(iv) Lower triangular matrices

(v) Zero matrix

A <u>vector</u> is a matrix with only one column. $\vec{v} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ component.

The set of all vectors with n components is denoted $\mathbb{R}^n$. vector space.



$$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Given a system of n linear equations in m variables:

$$\begin{matrix} a_{11}x_1 + \cdots + a_{1m}x_m = b_1 \\ \vdots \\ a_{n1}x_1 + \cdots + a_{nm}x_m = b_n \end{matrix} \xrightarrow[\text{matrix}]{\text{augmented}} \left[\begin{array}{ccc|c} a_{11} & \cdots & a_{1m} & b_1 \\ \vdots & & \vdots & \vdots \\ a_{n1} & \cdots & a_{nm} & b_n \end{array}\right]$$

we can simplify the augmented matrix using the following row operations:

(i) Divide a row by a non-zero scalar.

(ii) Subtract a multiple of a row from another row.

(iii) Swap two rows.

<u>Example:</u>

$$\begin{matrix} 2x + 8y + 4z = 2 \\ 2x + 5y + z = 5 \\ 4x + 10y - z = 1 \end{matrix} \longrightarrow \left[\begin{array}{ccc|c} 12 & 48 & 4 & 2 \\ 02 & -35\ 1 & 1 & 5 \\ 04 & 10 & -1 & 1 \end{array}\right]$$

1. Divide R_1 by 2.
2. Subtract R_1 from R_2 twice.
3. Subtract R_1 from R_3 four times.
4. Divide R_2 by -3.

$4-1-1-1-1=0$

$$\begin{matrix} x = 11 \\ y = -4 \\ z = 3 \end{matrix} \longleftarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

The simplified form of a matrix is called <u>reduced row-echelon form</u>:

"divide" ⇝ (i) If a row has a non-zero entry, then the first non-zero entry is a 1.
by constants

(called <u>leading 1</u> or <u>pivot</u>)

"subtract" ⇝ (ii) If a column contains a leading 1, then all other entries in the column are 0.
columns

"swap" ⇝ (iii) If a row contains a leading 1, each row above it contains a leading 1

further to the left.