Example:

$$\left[\begin{array}{ccc|c} 2 & 8 & 4 & 2 \\ 2 & 5 & 1 & 5 \\ 4 & 10 & -1 & 1 \end{array}\right] \xrightarrow{R_1 \cdot \frac{1}{2}} \left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 2 & 5 & 1 & 5 \\ 4 & 10 & -1 & 1 \end{array}\right] \xrightarrow[R_3 - 4 \cdot R_1]{R_2 - 2 \cdot R_1} \left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & -3 & -3 & 3 \\ 0 & -6 & -9 & -3 \end{array}\right]$$

$$\xrightarrow{R_2 \cdot \frac{1}{-3}} \left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & -6 & -9 & -3 \end{array}\right] \xrightarrow{R_1 - 4 \cdot R_2 \quad R_3 + 6 \cdot R_2} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -3 & -9 \end{array}\right]$$

$$\xrightarrow{R_3 \cdot \frac{1}{-3}} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_2 - R_3]{R_1 + 2 \cdot R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$x = 11$

$y = -4$

$z = 3$

Operations:

1. Multiply scalar.
2. Subtract rows from other rows
3. Swap rows

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

rref condition (iii): if we have a leading one on a row, every row above it has a leading one to the left.

Question: Is the zero matrix in rref?

No , Yes , No

(i) ✓ (ii) ✓ (iii) ✓

<u>Question:</u> (True or False) Every matrix in rref has a leading one.

<u>Question:</u> (True or False) Every non-zero matrix in rref has 1's in the diagonal.

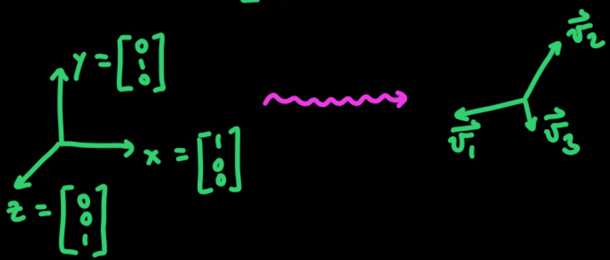
True , False , False

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{array}{c} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{l} L_1 \\ L_2 \\ L_3 \end{array}$$

$$\neq$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = [\vec{v}_1 \; \vec{v}_2 \; \vec{v}_3]$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$



one solution:
$$\left[\begin{array}{ccc|c} 1 & & & * \\ & 1 & & * \\ & & 1 & * \end{array}\right]$$

<u>no</u> solution
$$\left[\begin{array}{ccc|c} * & * & * & * \\ & & & \\ 0 & 0 & 0 & 1 \end{array}\right]$$ ← this row conveys 0 = 1.

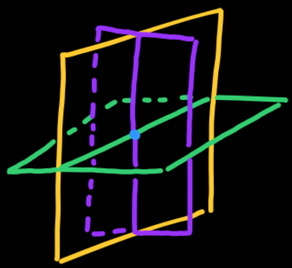
infinitely many
$$\left[\begin{array}{c|c} \quad & \quad \end{array}\right]$$ } <u>not</u> [0 0 0 | 1]

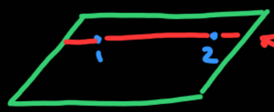
$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \end{array}\right]$$ ⟵ one equation does not matter, so one variable is automatically free.

One equation gives one constraint.

Living in $\mathbb{R}^n$, one equation gives an $(n-1)$-subspace.

A few $(n-1)$-subspaces intersect in zero, one, or infinitely many points.





the line between 1 and 2 is completely inside 

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$x + y = 1$ $\quad$ $y = 1 - x$ $\quad$ $\vec{v} = \begin{bmatrix} x \\ 1-x \end{bmatrix}$ $\quad$ x is a real number.

$x = x$

$y = 1 - x$

Problem 1.1.22:

x = # of female offspring

y = # of male offspring

Emile has twice as many sisters as brothers ⟶ $x = 2(y-1)$

Supposing Emile is male, he has $y-1$ brothers and x sisters.

Gertrude has y brothers and $x-1$ sisters ⟶ $y = x - 1$.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$x + y = 1$

$2x + 2y = 2$

$x + y + z = 1$

$2x + 3y + z = 1$

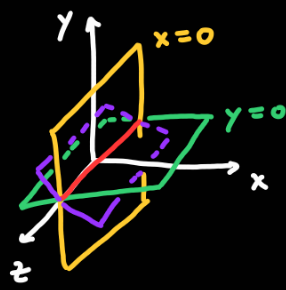
$0 = 0$

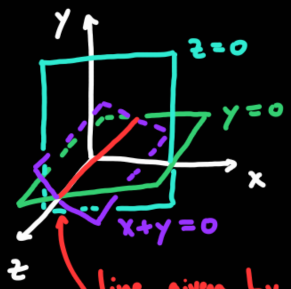
$0 = 1$

$x = 0$

$y = 0$

$x + y = 0$





line given by the z-axis.

$x + y = 0$

$y = 0$

$z = 0$

gives the intersection of with