Math 33A
Linear Algebra and Applications

Discussion for January 10-14, 2022

Problem 1.
   Show that if T is a linear transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$, then

$$T\begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = x_1 T(\vec{e_1}) + \cdots + x_m T(\vec{e_m}),$$

where $\vec{e_1}, \ldots, \vec{e_m}$ are the standard vectors in $\mathbb{R}^m$.

Problem 2.
   Describe all linear transformations from $\mathbb{R}$ to $\mathbb{R}$. What do their graphs look like?

Problem 3.
   Describe all linear transformations from $\mathbb{R}^2$ to $\mathbb{R}$. What do their graphs look like?

Problem 4.
   Consider two linear transformations $\vec{y} = T(\vec{x})$ and $\vec{z} = L(\vec{y})$, where T goes from $\mathbb{R}^m$ to $\mathbb{R}^p$ and L goes from $\mathbb{R}^p$ to $\mathbb{R}^n$. Is the transformation $\vec{z} = L(T(\vec{x}))$ linear as well?

Problem 5.
   Let
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}.$$
Find the matrix of the linear transformation $T(\vec{x}) = B(A\vec{x})$.