MATH 33A.

Instructor: Pablo S. Ocal socal@math.ucla.edu

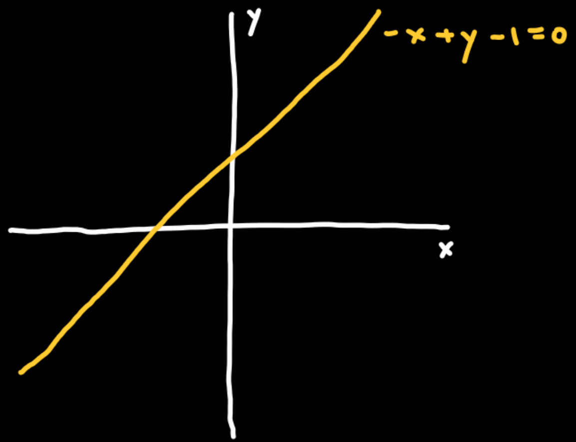
Syllabus: Grade 1 Grade 2 Grade 3

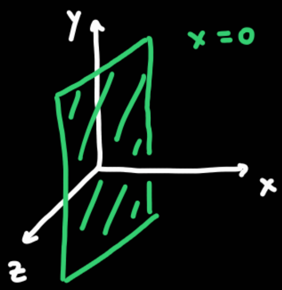
1. Introduction. (Chapter 1, Chapter 2)

Linear algebra is the study of <u>linear equations</u> and <u>linear transformations</u>.

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n - b = 0$$

coefficients variables constant term





Systems of linear equations have no solutions, one solution, or infinitely many solutions.

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n + b_1 = 0$$

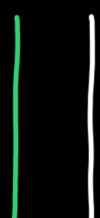
$$\vdots$$

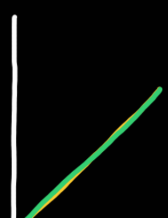
$$a_{n1} x_1 + a_{n2} x_2 + \cdots + a_{nn} x_n + b_n = 0$$

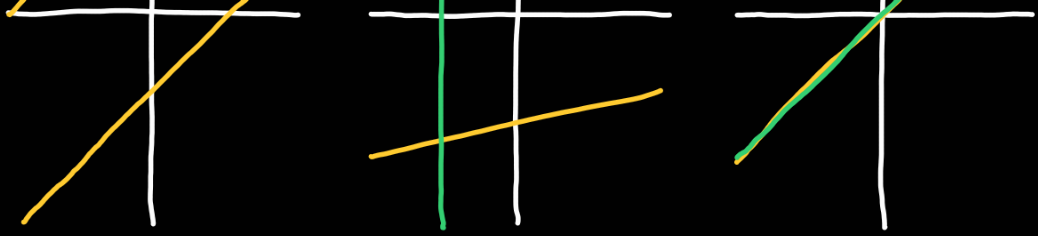
row

a_{ij}

column







Matrices, matrix:

rows

$$\begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & & \\ a_{n1} & \cdots & a_{nm} \end{bmatrix}$$ is an $n \times m$ with entries a_{ij}.

columns

A <u>matrix</u> is a rectangular array of numbers. If a matrix has n rows and m columns, it has <u>size</u> $n \times m$. We say that two matrices A, B are equal if their entries a_{ij}, b_{ij} coincide.

Some special families of matrices have names:

(i) Square matrices. $(n \times n)$

(ii) Diagonal matrices. (the only non-zero entries are a_{ii})

(iii) Upper triangular matrices. $\leftarrow$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

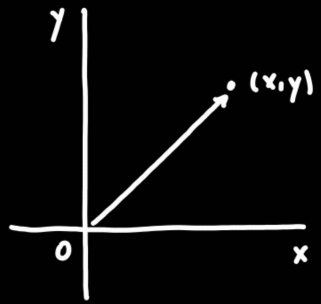
(iv) Lower triangular matrices.

(v) Zero matrix.

A <u>vector</u> is a matrix with only one column. $\vec{v} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ components

The set of all vectors with n entries is denoted by $\mathbb{R}^n$.

vector space



$$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Given a system of linear equations:

$$\begin{matrix} a_{11}x_1 + \cdots + a_{1m}x_m = b_1 \\ \vdots \\ a_{n1}x_1 + \cdots + a_{nm}x_m = b_n \end{matrix} \quad \xrightarrow[\text{matrix}]{\text{augmented}} \quad \left[\begin{array}{ccc|c} a_{11} & \cdots & a_{1m} & b_1 \\ \vdots & & \vdots & \vdots \\ a_{n1} & \cdots & a_{nm} & b_n \end{array}\right]$$

we simplify it using three row operations:

(1) Divide a row by a non-zero scalar.

(2) Subtract a multiple of a row from another one.

(3) Swap two rows.

<u>Example:</u>

$$\begin{matrix} 2x + 8y + 4z = 2 \\ 2x + 5y + z = 5 \\ 4x + 10y - z = 1 \end{matrix} \longrightarrow \left[\begin{array}{ccc|c} {\color{blue}1}2 & {\color{blue}4}8 & {\color{blue}2}4 & {\color{blue}1}2 \\ {\color{blue}0}2 & {\color{blue}-3}5 & {\color{blue}-3}1 & {\color{blue}3}5 \\ {\color{blue}0}4 & {\color{blue}-6}10 & {\color{blue}-9}-1 & {\color{blue}-3}1 \end{array}\right]$$

1. Divide R_1 by 2.
2. Subtract $2 \cdot R_1$ from R_2.
3. Subtract $4 \cdot R_1$ from R_3.

$$\begin{matrix} x = 11 \\ y = -4 \\ z = 3 \end{matrix} \longleftarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

The simplified form is called <u>reduced row-echelon form</u>:

(i) If a row has non-zero entries, then the first one is a 1.

(<u>leading</u> 1 or <u>pivot</u>)

(ii) If a column contains a leading 1, then all the other entries in the column are 0.

(iii) If a row contains a leading 1, then each row above it contains a leading 1 further to the left.

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & \boxed{2} \end{array}\right]$$

<u>not</u> rref!

← bottom row has non-zero entry, but does <u>not</u> have a leading 1.

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

<u>is</u> rref!