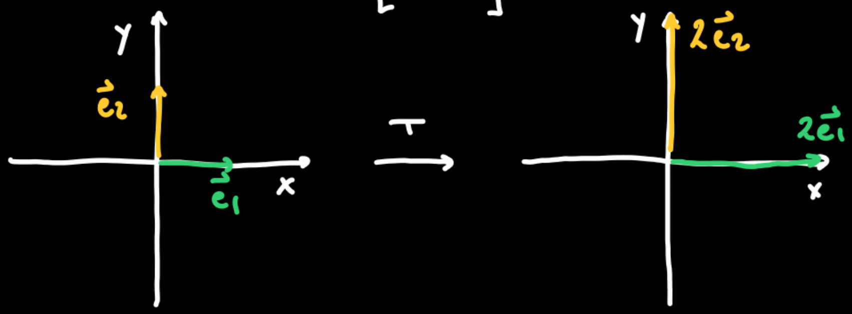
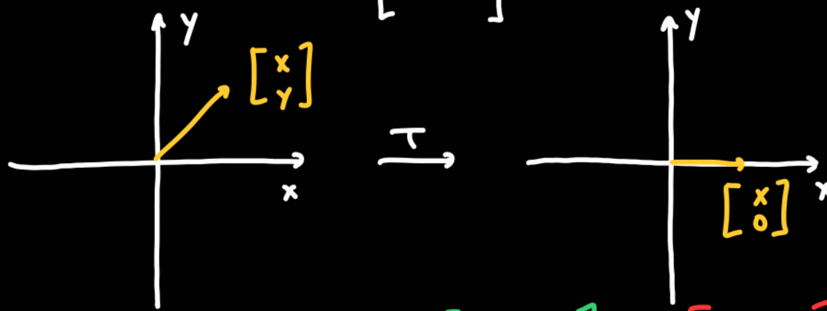
Example:

1. What does the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ do to $\mathbb{R}^2$?



2. What does the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ do to $\mathbb{R}^2$?



$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\mathbb{R}^2$

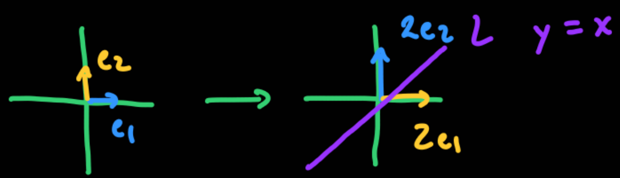
$\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \neq \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

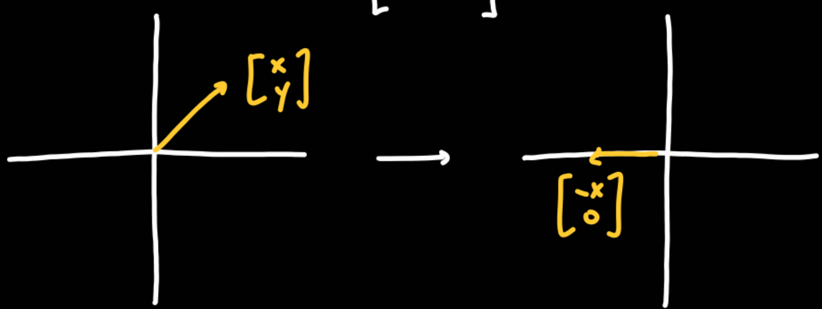
$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\mathbb{R}^3$

$\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\vec{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$



3. What does the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$ do to $\mathbb{R}^2$?



4. What does the matrix $\begin{bmatrix} 1 & -1 \\ \end{bmatrix}$ do to $\mathbb{R}^2$?

What does the matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ do to $\mathbb{R}^2$?

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\text{length } \sqrt{2}} = \underbrace{\begin{bmatrix} 0 \\ 2 \end{bmatrix}}_{\text{length } 2}$$

$\cdot\sqrt{2}$

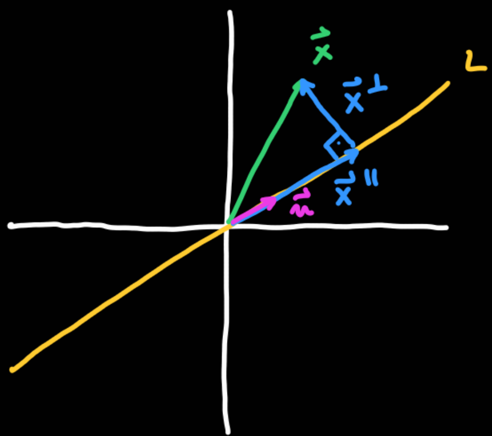
(i) scaling by $\sqrt{2}$.

(ii) rotating by $\frac{\pi}{4}$.

Scaling:

They are given by multiplication by $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, k in $\mathbb{R}$.

Orthogonal projections:



$$\vec{x} = \vec{x}^{\parallel} + \vec{x}^{\perp}$$

parallel to L orthogonal to L

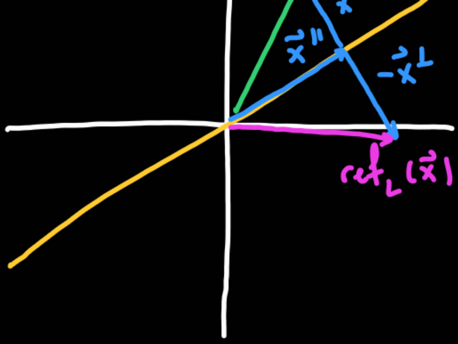
$$\text{proj}_L(\vec{x}) = \vec{x}^{\parallel} = (\vec{x}\cdot\vec{u})\,\vec{u}$$ <u>orthogonal projection</u>

The dot product $(\vec{x}\cdot\vec{u})$ is the length of $\vec{x}^{\parallel}$.

$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ unitary $\begin{bmatrix} u_1^2 & u_1u_2 \\ u_1u_2 & u_2^2 \end{bmatrix}$

Reflection:

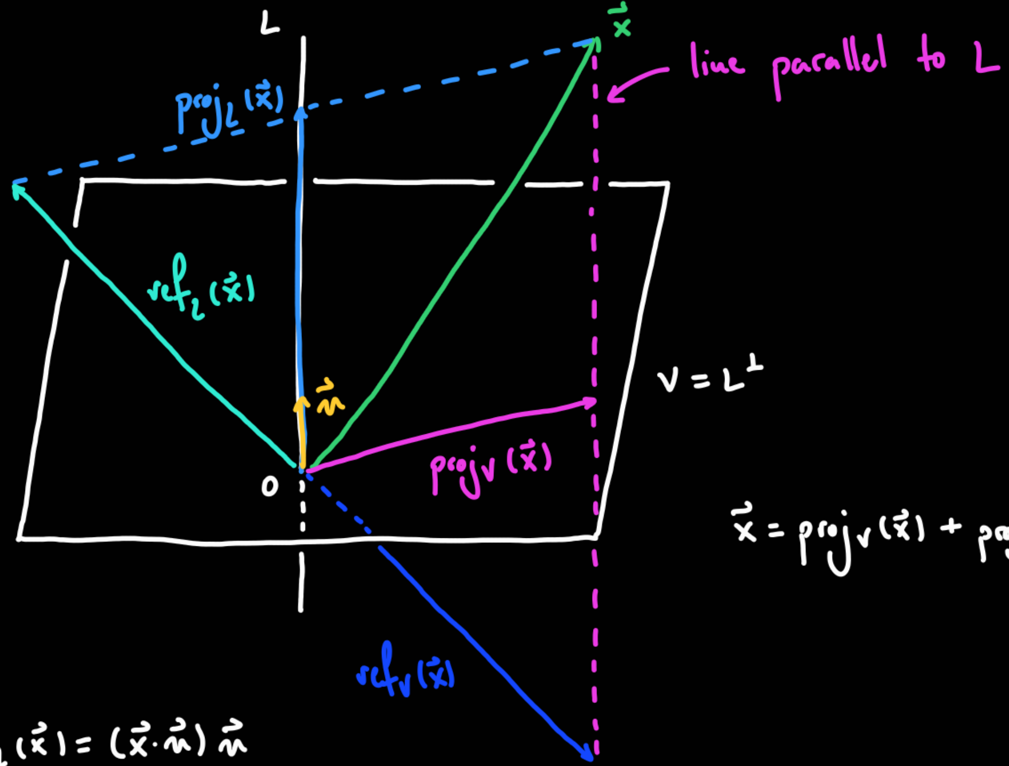
The reflection of $\vec{x}$ onto L is:



$$ref_L(\vec{x}) = \vec{x}^{\parallel} - \vec{x}^{\perp}.$$

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \text{ unitary} \qquad \begin{bmatrix} 2u_1^2 - 1 & 2u_1u_2 \\ 2u_1u_2 & 2u_2^2 - 1 \end{bmatrix}$$

Orthogonal projection and reflection in $\mathbb{R}^3$:



$$\vec{x} = proj_V(\vec{x}) + proj_L(\vec{x})$$

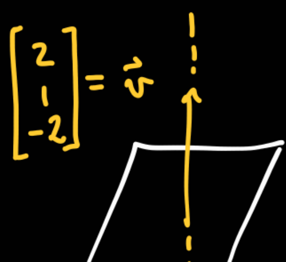
(i) $proj_L(\vec{x}) = (\vec{x} \cdot \vec{u})\,\vec{u}$

(ii) $proj_V(\vec{x}) = \vec{x} - proj_L(\vec{x})$

(iii) $ref_L(\vec{x}) = proj_L(\vec{x}) - proj_V(\vec{x})$

(iv) $ref_V(\vec{x}) = proj_V(\vec{x}) - proj_L(\vec{x})$

Example:



$V: \quad 2x_1 + x_2 - 2x_3 = 0$

$$\vec{x} : \begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix}$$

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{2^2+1^2+2^2}} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1/3 \\ -2/3 \end{bmatrix}$$

(i) $\text{proj}_L(\vec{x}) = (\vec{x} \cdot \vec{u})\,\vec{u} = \left(5 \cdot \frac{2}{3} + 4 \cdot \frac{1}{3} + (-2) \cdot \frac{(-2)}{3}\right) \cdot \begin{bmatrix} 2/3 \\ 1/3 \\ -2/3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -4 \end{bmatrix}$

(ii) $\text{proj}_V(\vec{x}) = \vec{x} - \text{proj}_L(\vec{x}) = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

(iii) $\text{ref}_L(\vec{x}) = \text{proj}_L(\vec{x}) - \text{proj}_V(\vec{x}) = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix}$

(iv) $\text{ref}_V(\vec{x}) = \text{proj}_V(\vec{x}) - \text{proj}_L(\vec{x}) = \begin{bmatrix} -3 \\ 0 \\ 6 \end{bmatrix}$