Problem 2.4.44.:

b) $$M_n = \begin{bmatrix} 1 & n+1 & 2n+1 & \cdots & (n-1)\cdot n+1 \\ 2 & n+2 & 2n+2 & \cdots & (n-1)\cdot n+2 \\ \vdots & \vdots & \vdots & & \vdots \\ n & n+n & 2n+n & \cdots & n^2 \end{bmatrix}$$

$$\begin{array}{ccc} 0\cdot n+1 & 1\cdot n+1 & (n-1)\cdot n+1 \\ 0\cdot n+2 & 1\cdot n+2 & (n-1)\cdot n+2 \\ \vdots & \vdots & \vdots \\ 0\cdot n+n & 1\cdot n+n & (n-1)\cdot n+n \\ & & \\ c_1 & c_2 & c_n \end{array}$$

Idea: for row i, subtract the first row.

$$\begin{array}{c} \\ R_2-R_1 \\ R_3-R_1 \\ \vdots \\ R_n-R_1 \end{array} \begin{bmatrix} 1 & n+1 & 2n+1 & \cdots & (n-1)\cdot n+1 \\ 1 & 1 & 1 & \cdots & 1 \\ 2 & 2 & 2 & \cdots & 2 \\ \vdots & \vdots & \vdots & & \vdots \\ n-1 & n-1 & n-1 & \cdots & n-1 \end{bmatrix} \begin{array}{c} R_1 \text{ stays} \\ R_2 \text{ stays} \\ \rightsquigarrow \\ R_3-2\cdot R_2 \\ R_4-3\cdot R_2 \\ \vdots \\ R_n-(n-1)\cdot R_2 \end{array} \begin{bmatrix} 1 & n+1 & 2n+1 & \cdots & (n-1)n+1 \\ 1 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 0 & & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & & 0 \end{bmatrix}$$

To finish this, we subtract R_2-R_1, then divide R_2 by $-n$, then $R_1-(n+1)R_2$.

Inverses of linear transformations:

$T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$, $S: \mathbb{R}^m \longrightarrow \mathbb{R}^p$

"To undo T then S, we first undo S, then undo T".

$T^{-1}S^{-1}$, this should be $(ST)^{-1}$

Curiosity: The set of all matrices of size $n \times m$ is exactly the same space as $\mathbb{R}^{n \cdot m}$.

$$\begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nm} \end{bmatrix} \longleftrightarrow \begin{bmatrix} a_{11} \\ \vdots \\ a_{n1} \\ a_{12} \\ \vdots \\ a_{n2} \\ \vdots \\ a_{1m} \\ \vdots \\ a_{nm} \end{bmatrix}$$

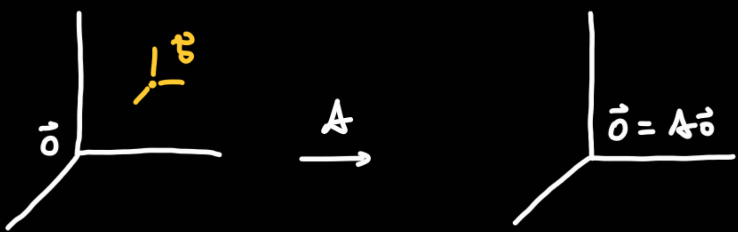
$n \times m$ matrix

vector with $n \cdot m$ entries

Lie group / Lie algebra

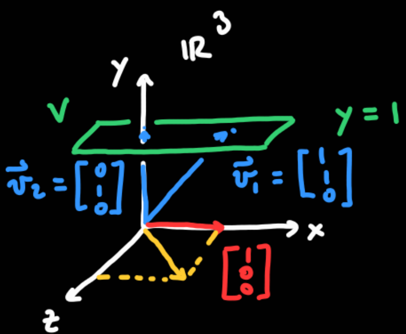
$\mathbb{R}^{n \cdot m}$

$$\begin{array}{ccc} \text{matrix} & & \text{vector} \\ \rightsquigarrow & & \rightsquigarrow \\ [a] & \longleftrightarrow & [a] \\ & & \mathbb{R} \end{array}$$



$A\vec{x} = \vec{b}$

"subspace" will always contain $\vec{0}$.



$$\vec{v}_1 - \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ \ldots \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ \ldots \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \ldots \end{bmatrix}$$

$\begin{bmatrix} 0 \end{bmatrix} \quad \begin{bmatrix} 1 \end{bmatrix} \quad \begin{bmatrix} -1 \end{bmatrix}$

"Translating up" is "artificially" making the vector $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ into the "origin" of r.

Problem 2.TF.1.:

$$\begin{bmatrix} 5 & 6 \\ -6 & 5 \end{bmatrix}$$

A linear transformation $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ is a rotation if and only if $a^2+b^2=1$.

A scaling is $\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$.

A rotation combined with a scaling is $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix} = \begin{bmatrix} a\cdot r & -b\cdot r \\ b\cdot r & a\cdot r \end{bmatrix}$.

So a rotation with a scaling satisfies $(ar)^2+(br)^2 = r^2\cdot(a^2+b^2) = r^2$.

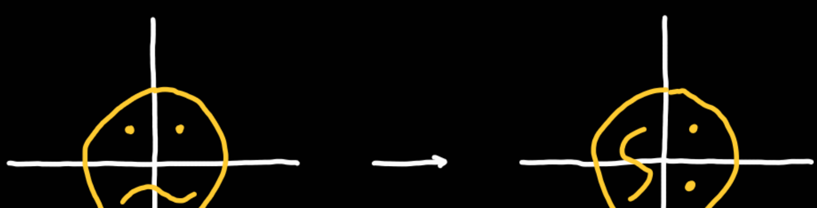
Now $5^2+(-6)^2 = 25+36 = 61$, so it is a rotation combined with a scaling of $\sqrt{61}$.

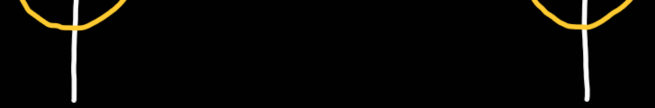
Theorem 2.2.4. in the book.

Problem 2.1.26.:

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ sends $\vec{e}_1$ to $\vec{e}_2$
$\vec{e}_2$ to $\vec{e}_1$.

This is a reflection about the line $y=x$.





Worksheet 2.3.:

A linear transformation from $\mathbb{R}^2 \to \mathbb{R}$ is a 1×2 matrix. So these are of the form $\vec{y} = \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, so $\vec{y} = a x_1 + b x_2$ with $\vec{y} = [y]$.

We have $y = ax_1 + bx_2$, a plane through the origin.