Recall: Inverse trig., hyperbolic, inverse hyperbolic.

Section 8.1.: Integration by parts.

This is just rewriting the differential of the product of two functions.

$$\frac{d}{dx}\big(f(x)\cdot g(x)\big) = \frac{df(x)}{dx}\cdot g(x) + f(x)\cdot\frac{dg(x)}{dx}$$

$$\text{Function} \underset{\int dx}{\overset{\frac{d}{dx}}{\rightleftarrows}} \text{Function}$$

Applying that derivatives and integrals are inverses of each other:

$$\int\left(\frac{d}{dx}\big(f(x)\cdot g(x)\big)\right) = \int\left(\frac{df(x)}{dx}\cdot g(x)\right) + \int\left(f(x)\cdot\frac{dg(x)}{dx}\right)$$

$$\underline{f(x)\cdot g(x)} = \int\big(\underline{f'(x)\cdot g(x)}\big) + \int\big(\underline{f(x)\cdot g'(x)}\big)$$

Integration by parts:

$$\boxed{\int \underline{f(x)\cdot g'(x)\,dx} = \underline{f(x)\cdot g(x)} - \int \underline{f'(x)\cdot g(x)\,dx}}$$

$f(x) = u$

$g(x) = v$

$$\boxed{\int u\cdot dv = u\cdot v - \int v\cdot du}$$

Example:

$$\int x\cdot\cos(x)\,dx \underset{IP}{=} x\cdot\sin(x) - \int 1\cdot\sin(x)\,dx = x\cdot\sin(x) + \cos(x) + C$$

$f(x) = x$ $\qquad$ $f'(x) = 1$

$g'(x) = \cos(x) \rightsquigarrow g(x) = \sin(x)$

integrate

Guidelines for choosing u and v:

$$\boxed{\int u\cdot dv = u\cdot v - \int v\cdot du}$$

polynomials 1. We want du to be simpler than u. $\left(\frac{df(x)}{dx}\text{ simpler than } f(x)\right)$.

exponentials 2. We need to know how to compute $\int dv = v$. $\left(\int \frac{dg(x)}{dx} = g(x)\right)$.

Example:

① $\int x \cdot e^x \, dx = x \cdot e^x - \int e^x \, dx = x \cdot e^x - e^x + c_1$.

$u = x \qquad du = 1$

$dv = e^x \qquad v = e^x$

$$\int u \, dv = uv - \int v \, du$$

$\int x \cdot e^x \, dx = e^x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot e^x \, dx$

$u = e^x \qquad du = e^x$

$dv = x \qquad v = \frac{x^2}{2}$

② $\int x^2 \cdot \cos(x) \, dx = x^2 \cdot \sin(x) - \int \sin(x) \cdot 2 \cdot x \cdot dx =$

$u = x^2 \qquad du = 2x \qquad\qquad u = x \qquad du = 1$

$dv = \cos(x) \qquad v = \sin(x) \qquad\qquad dv = \sin(x) \qquad v = -\cos(x)$

$= x^2 \cdot \sin(x) - 2 \cdot \left(x \cdot (-\cos(x)) - \int (-\cos(x)) \cdot 1 \cdot dx \right) =$

$= x^2 \cdot \sin(x) + 2x \cdot \cos(x) - 2 \cdot \int \cos(x) \, dx =$

$= x^2 \cdot \sin(x) + 2x \cdot \cos(x) - 2 \cdot \sin(x) + c_1$.

③ $\int e^x \cdot \cos(x) \, dx = e^x \cdot \cos(x) - \int e^x \cdot (-\sin(x)) \, dx = e^x \cdot \cos(x) + e^x \cdot \sin(x) - \int e^x \cdot \cos(x) \, dx$

$u = \cos(x) \qquad du = -\sin(x) \qquad\qquad u = \sin(x) \qquad du = \cos(x)$

$dv = e^x \qquad v = e^x \qquad\qquad dv = e^x \qquad v = e^x$

$$\int u \, dv = uv - \int v \, du$$

$2 \cdot \int e^x \cdot \cos(x) \, dx = e^x \cdot \left(\cos(x) + \sin(x) \right)$

$$\int e^{x}\cdot\cos(x)\,dx = \frac{e^{x}}{2}\cdot(\cos(x)+\sin(x)) + c_1$$

Bessel integrals.

Exercise: Integrate by parts: $\int x\cdot x\cdot dx$

Integration by parts for definite integrals: $\int_a^b u\,dv = u\,v\Big|_a^b - \int_a^b v\,du$

Example: Integrate:

① $\int_1^3 \ln(x)\cdot 1\cdot dx = \ln(x)\cdot x\Big|_1^3 - \int_1^3 x\cdot\frac{1}{x}\,dx = x\cdot\ln(x)\Big|_1^3 - \int_1^3 dx =$

$u=\ln(x)$ $\quad du=\frac{1}{x}$

$dv=1$ $\quad v=x$

$= x\cdot\ln(x)\Big|_1^3 - x\Big|_1^3 = (3\cdot\ln(3)-0)-(3-1) = 3\cdot\ln(3)-2.$

② $\int x^{n}\cdot e^{x}\,dx = x^{n}\cdot e^{x} - \int n\cdot x^{n-1}\cdot e^{x}\,dx = x^{n}\cdot e^{x} - n\cdot\int \overline{x^{n-1}}\cdot e^{x}\,dx$

n natural number

$u=x^{n}$ $\quad du=n\cdot x^{n-1}$

$dv=e^{x}$ $\quad v=e^{x}$

$$\int u\,dv = u\,v - \int v\,du$$

Question: There is a real number x such that $\sin(x)=2$. T / F.

The derivative of e^{x} can be negative. T / F.