Math 33A
Linear Algebra and Applications

Discussion for June 27-July 1, 2022

Problem 1.

We say that two $n \times m$ matrices in reduced row-echelon form are of the same type if they contain the same number of leading 1's in the same positions. Give an example of two 2×3 matrices of the same type. Give an example of two 2×3 matrices of different type.

Problem 2($\star$).

How many types of 2×2 matrices in reduced row-echelon form are there?

Problem 3.

How many types of 3×2 matrices in reduced row-echelon form are there?

Problem 4.

Suppose you apply Gauss–Jordan elimination to a matrix. Explain how you can be sure that the resulting matrix is in reduced row-echelon form.

Problem 5.

Suppose matrix A is transformed into matrix B by means of an elementary row operation. Is there an elementary row operation that transforms B into A? Explain.

Problem 6.

Suppose matrix A is transformed into matrix B by a sequence of elementary row operations. Is there a sequence of elementary row operations that transforms B into A? Explain.

Problem 7.

Consider an $n \times m$ matrix A. Can you transform $\text{rref}(A)$ into A by a sequence of elementary row operations? Explain.

Problem 8.

Show that if T is a linear transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$, then

$$T\begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = x_1 T(\vec{e_1}) + \cdots + x_m T(\vec{e_m}),$$

where $\vec{e_1}, \ldots, \vec{e_m}$ are the standard vectors in $\mathbb{R}^m$.

Problem 9($\star$).

Describe all linear transformations from $\mathbb{R}$ to $\mathbb{R}$. What do their graphs look like?

Problem 10.

Describe all linear transformations from $\mathbb{R}^2$ to $\mathbb{R}$. What do their graphs look like?

Problem 11.

Consider two linear transformations $\vec{y} = T(\vec{x})$ and $\vec{z} = L(\vec{y})$, where T goes from $\mathbb{R}^m$ to $\mathbb{R}^p$ and L goes from $\mathbb{R}^p$ to $\mathbb{R}^n$. Is the transformation $\vec{z} = L(T(\vec{x}))$ linear as well?

Problem 12.

Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}.$$

Find the matrix of the linear transformation $T(\vec{x}) = B(A\vec{x})$.