Problem 2.1.14:

a) $\begin{bmatrix} 2 & 3 \\ 5 & k \end{bmatrix}$ Problem 13: a 2x2 matrix is invertible if and only if $ad-bc \neq 0$.

$$2k - 5\cdot 3 \neq 0 \rightsquigarrow k \neq \frac{15}{2}.$$

b) $\begin{bmatrix} 2 & 3 \\ 5 & k \end{bmatrix}^{-1} \underset{\uparrow}{=} \frac{1}{2k-15}\begin{bmatrix} k & -5 \\ -3 & 2 \end{bmatrix}$, we want all the entries to be integers.

Problem 13

So $k \neq \frac{15}{2}$.

$\dfrac{k}{2k-15}$ so $k = x_1 \cdot (2k-15)$. This is a system of equations. Does it

$\dfrac{-5}{2k-15}$ $\quad -5 = x_2 \cdot (2k-15) \longrightarrow \dfrac{-5}{x_2} = 2k-15 \quad k = \dfrac{15}{2} - \dfrac{5}{2\cdot x_2}$ have any

$\dfrac{-3}{2k-15}$ $\quad -3 = x_3 \cdot (2k-15) \quad k = \dfrac{15}{2} - \dfrac{3}{2\cdot x_3}$ solutions?

$\dfrac{2}{2k-15}$ $\quad 2 = x_4 \cdot (2k-15) \quad k = \dfrac{15}{2} - \dfrac{2}{2\cdot x_3}$

Since 2, 3, 5 are prime, we must have $x_2 = x_3 = x_4 = 1$. ⚠ only true for k integers!

Row-reduced echelon form:

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \longleftrightarrow \begin{aligned} x+2y &= 0 \\ z &= 0 \\ 0 &= 0 \end{aligned} \qquad \begin{aligned} x &= -2y \\ z &= 0 \\ 0 &= 0 \end{aligned}$$

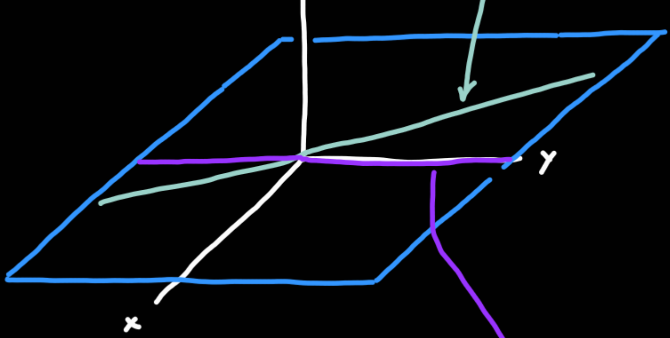
$\longleftarrow$ x is determined by y

$\longleftarrow$ z is determined

y is a free variable

free variable

$$\begin{bmatrix} -2y \\ y \\ 0 \end{bmatrix}$$



$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \longleftrightarrow \begin{aligned} x &= 0 \\ z &= 0 \\ 0 &= 0 \end{aligned} \qquad \begin{aligned} x &= 0 \\ z &= 0 \\ 0 &= 0 \end{aligned}$$

$$\begin{bmatrix} 0 \\ y \\ 0 \end{bmatrix}$$