Problem 2.1.49:

A transition matrix $\quad A = \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_m \\ | & & | \end{bmatrix} \quad \vec{v}_j = \begin{bmatrix} a_{1j} \\ \vdots \\ a_{nj} \end{bmatrix} \quad j = 1, \dots, m \, , \quad \sum_{i=1}^{n} a_{ij} = 1$

$\vec{x}$ distribution vector $\quad \vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} \quad x_1 + \cdots + x_m = 1 = \sum_{i=1}^{m} x_i$

$$A\vec{x} = \vec{v}_1 \cdot x_1 + \cdots + \vec{v}_m \cdot x_m = \begin{array}{c} \\ i \\ \\ \end{array} \begin{bmatrix} \\ a_{i1}x_1 + \cdots + a_{im}x_m \\ \\ \end{bmatrix}$$

for this to be a distribution vector we need:

$$\sum_{i=1}^{n} (a_{i1}x_1 + \cdots + a_{im}x_m) = 1$$

check that this is true.

$$\sum_{i=1}^{n} (a_{i1}x_1 + \cdots + a_{im}x_m) = \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij}\, x_j = \sum_{j=1}^{m} \sum_{i=1}^{n} a_{ij} x_j =$$

$$= \sum_{j=1}^{m} x_j \underbrace{\left(\sum_{i=1}^{n} a_{ij} \right)}_{1} = \underbrace{\sum_{j=1}^{m} x_j}_{1} = 1$$

A transition $\qquad$ $\vec{x}$ distribution

So $A\vec{x}$ is a distribution vector.

$$\begin{aligned} \sum_{i=1}^{n} (a_{i1}x_1 + \cdots + a_{im}x_m) = {} & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m + \\ & a_{21}x_1 + a_{22}x_2 + \cdots + a_{2m}x_m + \\ & \cdots \\ & a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nm}x_m = \end{aligned}$$

first column of A $\quad = x_1 \cdot (\underline{a_{11} + a_{21} + \cdots + a_{n1}}) +$

second column of A $\quad x_2 \cdot (\underline{a_{12} + a_{22} + \cdots + a_{n2}}) +$

$\cdots$

last column of A $\quad x_m \cdot (\underline{a_{1m} + a_{2m} + \cdots + a_{nm}}) =$

$$= x_1 + x_2 + \cdots + x_m = 1$$