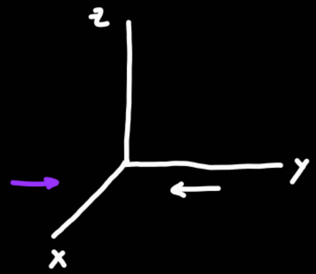
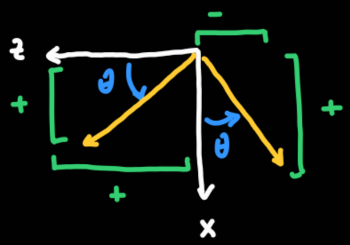
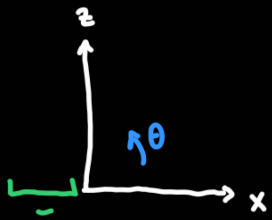
Midterm 1 Problem 4:

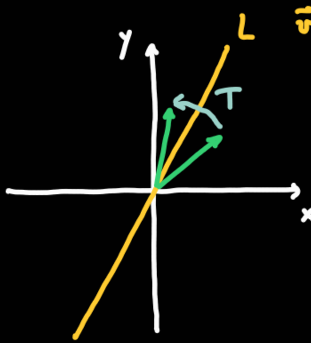




$$\begin{array}{cc} & x \\ x & \multirow{3}{*}{$\begin{bmatrix}\cos(\theta)\\0\\-\sin(\theta)\end{bmatrix}$} \\ y & \\ z & \end{array} \qquad \begin{array}{c} z \\ \begin{bmatrix}\sin(\theta)\\0\\\cos(\theta)\end{bmatrix}\end{array}$$



Problem 3.4.38.:



$\vec{v} = \begin{bmatrix}2\\3\end{bmatrix}$

T reflection about L

Want: a basis $\mathfrak{B}$ of $\mathbb{R}^2$ such that the matrix associated to T in $\mathfrak{B}$ is diagonal.

$$B = \begin{bmatrix} * & 0 \\ 0 & * \end{bmatrix} = \begin{bmatrix} [T(\vec{v}_1)]_{\mathfrak{B}} & [T(\vec{v}_2)]_{\mathfrak{B}} \end{bmatrix}$$

We want $T(\vec{v}_1)$ to be a multiple of $\vec{v}_1$.
$T(\vec{v}_2)$ $\vec{v}_2$.

Candidates: a vector in L and a vector in $L^\perp$.

$\mathrm{ref}_L(\vec{x}) = \vec{x} - 2\vec{x}^\perp$

$\vec{v}_1 = \begin{bmatrix}2\\3\end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix}-3\\2\end{bmatrix}$

$\mathfrak{B} = \{\vec{v}_1, \vec{v}_2\}$

$T(\vec{v}_1) = \vec{v}_1$

$T(\vec{v}_2) = -\vec{v}_2$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Problem 3.4.65.:

A is similar to B if there is an invertible S such that $B = S^{-1}AS$.

(a) We want to prove that A is similar to A. (A is $n \times n$)

We need to find an invertible S such that $A = S^{-1} A S$.

Take $S = I_n$.

(b) If A is similar to B, we want to prove that B is similar to A.

We know that there is an invertible matrix S such that $B = S^{-1} A S$.

We want an invertible T such that $A = T^{-1} B T$.

$(S^{-1})^{-1} = S$ so $S \cdot (S^{-1}) = I_n$

Since $B = S^{-1} A S$, multiplying by S on the left gives:

$SB = S(S^{-1} A S)$ so $SB = (S S^{-1}) A S$ so $SB = I_n A S$

so $SB = AS$.

Multiplying by S^{-1} on the right gives:

$SBS^{-1} = (AS) S^{-1} = A (S S^{-1}) = A I_n = A$. so $A = S B S^{-1}$.

Recall that we want $A = T^{-1} B T$. Take (i.e. define) $T = S^{-1}$. Now:

$(S^{-1})^{-1} B S^{-1} = S B S^{-1} = A$.

Since S is invertible, S^{-1} is also invertible.